Data Mining:
Mining Association Rules
Examples

Course Enrollments

Itemset. $I = \{$ CSC365, CSC366, CSC402, CSC405, CSC416, CSC454, CSC456, CSC464, CSC465, CSC471, CSC474, CSC480$\}$.

Column	Course Number	Course
1	CSC365	Intro Databases
2	CSC366	Database Design, Modeling, Implementation
3	CSC402	Software Requirements
4	CSC405	Software Construction
5	CSC416	Autonomous Mobile Robotics
6	CSC454	Implementation of OS
7	CSC456	Intro Computer Security
8	CSC464	Intro Networks
9	CSC465	Advanced Networks
10	CSC471	Intro Graphics
11	CSC474	Computer Animation
12	CSC480	Artificial Intelligence

Market Baskets. The market baskets in our dataset consist of the Computer Science electives selected by individual students. Consider the list of 20 market baskets in Figure 1

This list can be represented as a **full binary matrix** as shown in Figure 2.

Example 1. Consider the itemset $T = \{\mathsf{CSC365}, \mathsf{CSC366}, \mathsf{CSC416}\}$. Support set of T in the dataset is $Sup(T) = \{s_5, s_{18}\}$. Therefore,

$$support(T) = \frac{|Sup(T)|}{20} = \frac{2}{20} = 0.1.$$

s_1	CSC365, CSC366, CSC402, CSC405, CSC464
s_2	CSC402, CSC405, CSC454, CSC456, CSC480
s_3	CSC365, CSC416, CSC454, CSC464
s_4	CSC365, CSC366, CSC471, CSC474
s_5	CSC365, CSC366, CSC416, CSC471, CSC474, CSC480
s_6	CSC402, CSC405, CSC480
s_7	CSC416, CSC454, CSC456, CSC464, CSC465, CSC480
s_8	CSC456, CSC464, CSC465
s_9	CSC471, CSC474
s_{10}	CSC365, CSC456, CSC464, CSC471, CSC480
s_{11}	CSC416, CSC456, CSC464, CSC480
s_{12}	CSC365, CSC366, CSC402, CSC480
s_{13}	CSC365, CSC402, CSC405, CSC464
s_{14}	CSC402, CSC471, CSC480
s_{15}	CSC365, CSC366, CSC456, CSC464, CSC465
s_{16}	CSC471, CSC474, CSC480
s_{17}	CSC454, CSC471
s_{18}	CSC365, CSC366, CSC416, CSC480
s_{19}	CSC402, CSC405, CSC471, CSC474
s_{20}	CSC454, CSC480

Figure 1: Student Enrollment Dataset: Market Baskets

Item	365	366	402	405	416	454	456	464	465	471	474	480
s_1	1	1	1	1	0	0	0	1	0	0	0	0
s_2	0	0	1	1	0	1	1	0	0	0	0	1
s_3	1	0	0	0	1	1	1	0	0	0	0	0
s_4	1	1	0	0	0	0	0	0	0	1	1	0
s_5	1	1	0	0	1	0	0	0	0	1	1	1
s_6	0	0	1	1	0	0	0	0	0	0	0	1
s_7	0	0	0	0	1	1	1	1	1	0	0	1
s_8	0	0	0	0	0	1	0	1	1	0	0	0
s_9	0	0	0	0	0	0	0	0	0	1	1	0
s_{10}	1	0	0	0	0	0	1	1	0	1	0	1
s_{11}	0	0	0	0	1	0	1	1	0	0	0	1
s_{12}	1	1	1	0	0	0	0	0	0	0	0	1
s_{13}	1	0	1	1	0	0	0	0	0	0	0	1
s_{14}	0	0	1	0	0	0	0	0	0	1	0	1
s_{15}	1	1	0	0	0	0	1	1	1	0	0	0
s_{16}	0	0	0	0	0	0	0	0	0	1	1	1
s_{17}	0	0	0	0	0	1	0	0	0	1	0	0
s_{18}	1	1	0	0	1	0	0	0	0	0	0	1
s_{19}	0	0	1	1	0	0	0	0	0	1	1	0
s_{20}	0	0	0	0	0	1	0	0	0	0	0	1
Count:	9	6	7	5	5	6	6	6	3	8	5	12
Support:	0.45	0.3	0.35	0.25	0.25	0.3	0.3	0.3	0.15	0.4	0.25	0.6

Figure 2: Student Enrollment Dataset: Full Binary Vectors

Example 2. Consider an association rule $R_1 = \text{CSC402} \longrightarrow \text{CSC405}$.

The support set for R_1 is $Sup(R_1) = \{s_1, s_2, s_6, s_{13}, s_{19}\}$. The support of R_1 is

$$support(R_1) = \frac{|Sup(R_1)|}{20} = \frac{5}{20} = 0.25.$$

The support set for $\{\text{CSC402}\}$ is $\{s_1, s_2, s_6, s_{12}, s_{13}, s_{14}, s_{19}\}$. The confidence of the rule R_1 is then

$$confidence(R_1) = \frac{support(R_1)}{support(\{\text{CSC402}\})} = \frac{0.25}{0.35} = \frac{5}{7} = 0.714.$$

Apriori Algorithm

minConf. Consider the value of minimal support, minSup = 0.25.

Goal. We trace the work of the Apriori algorithm in discovery of frequent itemsets with support of at least minSup (0.25).

Step 1. Itemsets of size 1. First, we discover frequent itemsets of size 1.

$$F_1 = \{\{\text{CSC365}\}, \{\text{CSC366}\}, \{\text{CSC402}\}, \{\text{CSC405}\}, \{\text{CSC416}\}, \{\text{CSC454}\}, \{\text{CSC456}\}, \{\text{CSC464}\}, \{\text{CSC471}\}, \{\text{CSC474}\}, \{\text{CSC480}\}\}.$$

Note: $support(\{\text{CSC465}\}) = 0.15 <$ minSup, so CSC465 is excluded from consideration. All other columns have support of 0.25 or higher and they are included.

Step 2.1. Itemsets of size 2. Join Step. On this step, we construct the list of all *pairs* of items from C_1.

Note: The **join step** for size 2 itemsets is *trivial*: it involves computing cartesian product of C_1.

$$C_2 = F_1 \times F_1.$$

Step 2.2. Itemsets of size 2. Pruning Step. For itemsets of size 2, the pruning step of the cadidateGen() function is trivial. Nothing is pruned, C_2 remains intact.

Step 2.3. Itemsets of size 2. Support computation. Step 2.1 generated $\frac{11 \cdot 10}{2} = 55$ possible pairings. We now need to prune this set, by excluding from it all pairs of courses that have low support. We can construct the following Support table for our dataset:

	365	366	402	405	416	454	456	464	471	474	480
365	—	**6**	2	1	2	0	2	2	3	2	**5**
366		—	2	1	2	0	1	2	2	2	3
402			—	**5**	0	1	1	1	2	1	**5**
405				—	0	1	1	1	1	1	3
416					—	2	3	2	1	1	4
454						—	3	2	1	0	3
456							—	4	1	0	4
464								—	1	0	3
471									—	**5**	4
474										—	2
480											—

From the table above, the following pairs of courses exceed minsup:

Itemset	Baskets	Frequency	support
{CSC365, CSC366}	$\{s_1, s_4, s_5, s_{12}, s_{15}, s_{18}\}$	6	0.3
{CSC365, CSC480}	$\{s_5, s_{10}, s_{12}, s_{13}, s_{18}\}$	5	0.25
{CSC402, CSC405}	$\{s_1, s_2, s_6, s_{13}, s_{19}\}$	5	0.25
{CSC402, CSC480}	$\{s_2, s_6, s_{12}, s_{13}, s_{14}\}$	5	0.25
{CSC471, CSC474}	$\{s_4, s_5, s_9, s_{16}, s_{19}\}$	5	0.25

So, $F_2 = \{$ {CSC365, CSC366}, {CSC365, CSC480}, {CSC402, CSC405}, {CSC402, CSC480}, {CSC471, CSC474} $\}$.

Step 3.1. Itemsets of size 3. Join Step. On this step, we join all pairs of sets from F_2 trying to form candidate frequent itemsets of size 3. We are able to join the following pairs of sets:

First itemset	Second itemset	Join	ID
{<u>CSC365</u>, CSC366}	{<u>CSC365</u>, CSC480}	{CSC365, CSC366, CSC480}	c_1
{<u>CSC402</u>, CSC405}	{<u>CSC402</u>, CSC480}	{CSC402, CSC405, CSC480}	c_2
{CSC365, <u>CSC480</u>}	{CSC402, <u>CSC480</u>}	{CSC365, CSC402, CSC480}	c_3

$C_3 = \{$ {CSC365, CSC366, CSC480}, {CSC402, CSC405, CSC480}, {CSC365, CSC402, CSC480} $\}$.

Step 3.2. Itemsets of size 3. Pruning Step. For {CSC365, CSC366, CSC480}:

{CSC365, CSC366} $\in F_2$
{CSC365, CSC480} $\in F_2$
{CSC366, CSC480} $\notin F_2$

For {CSC402, CSC405, CSC480}:

{CSC402, CSC405} $\in F_2$
{CSC402, CSC480} $\in F_2$
{CSC405, CSC480} $\notin F_2$

For {CSC365, CSC402, CSC480}:

{CSC365, CSC480} $\in F_2$
{CSC402, CSC480} $\in F_2$
{CSC365, CSC402} $\notin F_2$

Therefore, all three elements of C_3 are **not frequent itemsets** and the **Apriori Algoritm** can stop there and return $F = F_1 \cup F_2$ as the set of all frequent itemsets.

Takehome Problem

Run **Apriori Algorithm** by hand with minSup = 0.2.

Generation of Association Rules

Frequent Itemsets. In previous section, we discovered that Student Enrollment dataset has 11 (eleven) frequent itemsets of size 1 (all singleton sets except for $\{\mathsf{CSC465}\}$) and five frequent itemsets of size 2:

$$\{\{\mathsf{CSC365}, \mathsf{CSC366}\}, \{\mathsf{CSC365}, \mathsf{CSC480}\}, \{\mathsf{CSC402}, \mathsf{CSC405}\}, \{\mathsf{CSC402}, \mathsf{CSC480}\}, \{\mathsf{CSC471}, \mathsf{CSC474}\}\}.$$

This gives rise to 10 candidate association rules with a single item on the right side. For each of them, we compute confidence.

ID	Rule	Frequent Itemset Support	Left side support	Confidence
R_1	CSC365 $\longrightarrow$ CSC366	$\frac{6}{20}$	$\frac{9}{20}$	$\frac{2}{3} = 0.667$
R_2	CSC366 $\longrightarrow$ CSC365	$\frac{6}{20}$	$\frac{6}{20}$	1
R_3	CSC365 $\longrightarrow$ CSC480	$\frac{5}{20}$	$\frac{9}{20}$	$\frac{5}{9} = 0.555$
R_4	CSC480 $\longrightarrow$ CSC365	$\frac{5}{20}$	$\frac{12}{20}$	$\frac{5}{12} = 0.41667$
R_5	CSC402 $\longrightarrow$ CSC405	$\frac{5}{20}$	$\frac{7}{20}$	$\frac{5}{7} = 0.714$
R_6	CSC405 $\longrightarrow$ CSC402	$\frac{5}{20}$	$\frac{5}{20}$	1
R_7	CSC402 $\longrightarrow$ CSC480	$\frac{5}{20}$	$\frac{7}{20}$	$\frac{5}{7} = 0.714$
R_8	CSC480 $\longrightarrow$ CSC402	$\frac{5}{20}$	$\frac{12}{20}$	$\frac{5}{12} = 0.41667$
R_9	CSC471 $\longrightarrow$ CSC474	$\frac{5}{20}$	$\frac{8}{20}$	$\frac{5}{8} = 0.625$
R_{10}	CSC474 $\longrightarrow$ CSC471	$\frac{5}{20}$	$\frac{5}{20}$	1

Depending on the values of minConf, we will report the following:

- minConf = 1. We report rules R_2, R_6 and R_{10}.
- $0.668 \leq$ minConf < 1. We report rules R_2, R_6 and R_{10} from above, plus R_5 and R_7.
- minConf > 0.5. In addition to the rules above, we report R_1, R_3 and R_9.

Takehome Problem

After discovering all frequent itemsets with support of at least 0.2, report all association rules in the dataset for minConf levels of 1, 0.75, 0.666 and 0.5.