Assignment #4: Subtyping

Overview

This assignment extends the type system implementation to support subtyping on record and function types.

Starting Point

Extend your solution to Assignment #3. There are no new types or terms to add. Instead you will update the typing for application (T-App) and if (T-If1, T-If2) to allow for subtyping.

Tests

Include in your submission a set of tests verifying that your implementation works as expected, both for terms that are well-typed and terms that are not.

Include a set of tests translated from the examples in the Ungraded Problem Set #4.

Typing Function

Extend your typing function to support subtyping in application (T-App) and if (T-If1, T-If2). For if, your implementation should support at least promoting the type of one branch to that of the other (in either direction, as indicated by T-If1, T-If2), but you can explore promoting to a common ancestor (the least upperbound) via a lattice join if you wish.

Grading

Grading will be divided as follows, and will be based on both functionality and quality of implementation.

Part	Percentage
Record Subtyping (S-Rcd)	35
Function Subtyping (S-Arrow)	35
Subtyping: Application (T-App)	10
Subtyping: If (T-If1, T-If2)	10
Tests	10

The terms for the expression language are to be inferred from the rules below coupled with the discussion in lecture (and in the textbook). Your implementation, of course, will work on the internal AST representation of such expressions, so you should be able to map the terms used in the typing rules to the variants declared in the Rust code.

nv is for numeric values.

$$\Gamma \vdash \text{true} : \text{Bool} \qquad \text{(T-True)}$$

$$\Gamma \vdash \text{false} : \text{Bool} \qquad \text{(T-False)}$$

$$\Gamma \vdash nv : \text{Int} \qquad \text{(T-IntConst)}$$

$$\frac{t_1 : Int}{\Gamma \vdash \text{iszero } t_1 : \text{Bool}} \qquad \text{(T-IsZero)}$$

$$\frac{\Gamma \vdash t_1 : \text{Int} \quad \Gamma \vdash t_2 : \text{Int}}{\Gamma \vdash t_1 + t_2 : \text{Int}} \qquad \text{(T-Add)}$$

$$\frac{\Gamma \vdash t_1 : \text{Int} \quad \Gamma \vdash t_2 : \text{Int}}{\Gamma \vdash t_1 - t_2 : \text{Int}} \qquad \text{(T-Sub)}$$

$$\frac{\text{x} : \tau \in \Gamma}{\Gamma \vdash \text{x} : \tau} \qquad \text{(T-Var)}$$

$$\frac{\Gamma, \text{x} : \alpha \vdash t_1 : \beta}{\Gamma \vdash \lambda \text{x} : \alpha.\ t_1 : \alpha \to \beta} \qquad \text{(T-Abs)}$$

$$\frac{\text{for each } i \quad \Gamma \vdash t_i : \tau_i}{\Gamma \vdash \{l_i = t_i\ ^{i \in 1..n}\} : \{l_i : \tau_i\ ^{i \in 1..n}\}} \qquad \text{(T-Rcd)}$$

$$\frac{\Gamma \vdash t_1 : \{l_i : \tau_i\ ^{i \in 1..n}\}}{\Gamma \vdash t_1.j : \tau_j} \qquad \text{(T-Proj)}$$

$$\frac{\Gamma \vdash t_j : \tau_j}{\Gamma \vdash \langle l_j = t_j \rangle \text{ as } \langle l_i : \tau_i\ ^{i \in 1..n} \rangle : \langle l_i : \tau_i\ ^{i \in 1..n} \rangle} \qquad \text{(T-Variant)}$$

$$\frac{\Gamma \vdash t_0 : \langle l_i : \tau_i\ ^{i \in 1..n} \rangle \quad \text{for each } i \quad \Gamma, x_i : \tau_i \vdash t_i : \tau}{\Gamma \vdash \text{case } t_0 \text{ of } \langle l_i = x_i \rangle \Rightarrow t_i\ ^{i \in 1..n} : \tau} \qquad \text{(T-Case)}$$

$$\Gamma \vdash \text{unit} : \text{Unit} \qquad \text{(T-Unit)}$$

$$\mathrm{S} <: \mathrm{S} \qquad \text{(S-Refl)}$$

$$\frac{\mathrm{S} <: \mathrm{U} \quad \mathrm{U} <: \mathrm{T}}{\mathrm{S} <: \mathrm{T}} \qquad \text{(S-Trans)}$$

$$\frac{\Gamma \vdash \mathrm{t} : \mathrm{S} \quad \mathrm{S} <: \mathrm{T}}{\Gamma \vdash \mathrm{t} : \mathrm{T}} \qquad \text{(S-Sub)}$$

$$\frac{\mathrm{P}_2 <: \mathrm{P}_1 \quad \mathrm{R}_1 <: \mathrm{R}_2}{\mathrm{P}_1 \to \mathrm{R}_1 <: \mathrm{P}_2 \to \mathrm{R}_2} \qquad \text{(S-Arrow)}$$

$$\frac{\{l_i \ ^{i\in 1..n}\} \subseteq \{k_j \ ^{j\in 1..m}\} \quad k_j = l_i \text{ implies } \alpha_j <: \beta_i}{\{k_j : \alpha_j \ ^{j\in 1..m}\} <: \{l_i : \beta_i \ ^{i\in 1..n}\}} \qquad \text{(S-Rcd)}$$

$$\frac{\Gamma \vdash t_1 : \mathrm{Bool} \quad \Gamma \vdash t_2 : \tau_1 \quad \Gamma \vdash t_3 : \tau_2 \quad \tau_2 <: \ \tau_1}{\Gamma \vdash \text{if } t_1 \text{ then } t_2 \text{ else } t_3 : \tau_1} \qquad \text{(T-If1)}$$

$$\frac{\Gamma \vdash t_1 : \mathrm{Bool} \quad \Gamma \vdash t_2 : \tau_1 \quad \Gamma \vdash t_3 : \tau_2 \quad \tau_1 <: \ \tau_2}{\Gamma \vdash \text{if } t_1 \text{ then } t_2 \text{ else } t_3 : \tau_2} \qquad \text{(T-If2)}$$

$$\frac{\Gamma \vdash t_1 : \alpha \to \beta \quad \Gamma \vdash t_2 : \alpha' \quad \alpha' <: \ \alpha}{\Gamma \vdash t_1 \ t_2 : \beta} \qquad \text{(T-App)}$$