Ungraded Problem Set #1

Overview

This problem set is meant to give you an idea of the scope of the questions that might appear on a quiz/exam.

1. Using the evaluation rules presented on the last page, evaluate each of the following showing each step. If the evaluation gets stuck, indicate this after the last step.
   - `iszero (1 + 4)`
   - `(1 + 4) + (2 - 3)`
   - `if true then 1 + false else true`
2. Discuss the implications of adding the following rule to the evaluation rules presented on the last page. More generally, what are the implications of such a rule in a language such as C or Java (where, technically, one can think of this `if` as an instance of the ternary operator)?

$$\frac{t_2 \longrightarrow t_2'}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \longrightarrow \text{if } t_1 \text{ then } t_2' \text{ else } t_3}$$

3. Indicate how the evaluation rules (from the last page) for addition and subtraction must change in order to require that the right operand is evaluated before the left operand. Specify any new rules and indicate the rules that are to be replaced.
4. Give the result of each of the following substitutions.
   - $[x \mapsto 2]\ (x+3)$
   - $[x \mapsto 2]\ (y+3)$
   - $[x \mapsto 2]\ (x+(\lambda y.\ x+y))$
   - $[x \mapsto 2]\ (x+((\lambda y.\ (x+y)+((\lambda x.\ x*2)y))x))$
5. Using the evaluation rules presented on the last page, evaluate each of the following showing each step. If the evaluation gets stuck, indicate this after the last step.
   - $(\lambda x\ .\ x+1)\ 2$
   - scc c_2
     where scc $= \lambda n.\ \lambda s.\ \lambda z.\ s\ (n\ s\ z)$
     and $\mathrm{c}_2 = \lambda s.\ \lambda z.\ s\ (s\ z)$
   - plus c_2 c_2
     where plus $= \lambda m.\ \lambda n.\ \lambda s.\ \lambda z.\ m\ s\ (n\ s\ z)$
     and $\mathrm{c}_2 = \lambda s.\ \lambda z.\ s\ (s\ z)$

The terms for the expression language are to be inferred from the rules below coupled with the discussion in lecture (and in the textbook).

nv is for numeric values (all integer values).

$$\text{if true then } t_2 \text{ else } t_3 \;\longrightarrow\; t_2 \qquad \text{(E-IfTrue)}$$

$$\text{if false then } t_2 \text{ else } t_3 \;\longrightarrow\; t_3 \qquad \text{(E-IfFalse)}$$

$$\frac{t_1 \longrightarrow t_1'}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \;\longrightarrow \text{if } t_1' \text{ then } t_2 \text{ else } t_3} \qquad \text{(E-If)}$$

$$\text{iszero } 0 \;\longrightarrow\; \text{true} \qquad \text{(E-IsZeroZero)}$$

$$\frac{nv_1 \neq 0}{\text{iszero } nv_1 \;\longrightarrow\; \text{false}} \qquad \text{(E-IsZeroNonZero)}$$

$$\frac{t_1 \longrightarrow t_1'}{\text{iszero } t_1 \;\longrightarrow\; \text{iszero } t_1'} \qquad \text{(E-IsZero)}$$

$$nv_1 + nv_2 \longrightarrow nv_1 +_{int} nv_2 \qquad \text{(E-AddConst)}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1 + t_2 \longrightarrow t_1' + t_2} \qquad \text{(E-AddLeft)}$$

$$\frac{t_2 \longrightarrow t_2'}{nv_1 + t_2 \longrightarrow nv_1 + t_2'} \qquad \text{(E-AddRight)}$$

$$nv_1 - nv_2 \longrightarrow nv_1 -_{int} nv_2 \qquad \text{(E-SubConst)}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1 - t_2 \longrightarrow t_1' - t_2} \qquad \text{(E-SubLeft)}$$

$$\frac{t_2 \longrightarrow t_2'}{nv_1 - t_2 \longrightarrow nv_1 - t_2'} \qquad \text{(E-SubRight)}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ t_2 \longrightarrow t_1'\ t_2} \qquad \text{(E-APP1)}$$

$$\frac{t_2 \longrightarrow t_2'}{v_1\ t_2 \longrightarrow v_1\ t_2'} \qquad \text{(E-APP2)}$$

$$(\lambda x.\ t_1)\ v_2 \longrightarrow [x \mapsto v_2]\ t_1 \qquad \text{(E-APPABS)}$$