Ungraded Problem Set #2

1. Using the typing rules presented on the last page, give a type derivation for each of the following assuming the specified type context (Γ) to start. If a type cannot be found for a term, clearly indicate where the derivation fails (show the derivation to that point).

   - `iszero 0`
     where $\Gamma = \{\}$
   - `succ (pred (succ 0))`
     where $\Gamma = \{\}$
   - `if true then succ 0 else true`
     where $\Gamma = \{\}$
   - `if x then succ 0 else pred 0`
     where $\Gamma = \{\text{x} : \text{Bool}\}$

2. Assume that addition and subtraction are added to the set of terms as in the last problem set (assuming now that all numeric values have type `Int` instead of `Nat`). Give a typing judgement for addition (i.e., complete the following).

$$\frac{}{\Gamma \vdash t_1 + t_2 :}$$

3. Using the typing rules presented on the last page, give a type derivation for each of the following assuming the specified type context (Γ) to start. If a type cannot be found for a term, clearly indicate where the derivation fails (show the derivation to that point).

   - $(\lambda \text{x} : \text{Nat} \text{ . succ x}) \ 0$
     where $\Gamma = \{\}$
   - $(\lambda \text{x} : \text{Nat} \text{ . succ x}) \ \text{true}$
     where $\Gamma = \{\}$
   - $(\lambda \text{x} : \text{Nat} \text{ . times x } 0) \ (\text{succ } 0)$
     where $\Gamma = \{\text{times} : \text{Nat} \rightarrow \text{Nat} \rightarrow \text{Nat}\}$
   - $(\lambda \text{x} : \text{Bool} \text{ . if x then } (\lambda \text{x} : \text{Nat} \text{ . succ } x) \ 0 \text{ else } (\lambda \text{x} : \text{Nat} \rightarrow \text{Bool} \text{ . x } 0) \ (\lambda \text{y} : \text{Nat} \text{ . x})) \ \text{true}$
     where $\Gamma = \{\text{x} : \text{Nat}\}$

The terms for the expression language are to be inferred from the rules below coupled with the discussion in lecture (and in the textbook).

$$\Gamma \vdash \text{true} : \text{Bool} \qquad \text{(T-True)}$$

$$\Gamma \vdash \text{false} : \text{Bool} \qquad \text{(T-False)}$$

$$\frac{\Gamma \vdash t_1 : \text{Bool} \quad \Gamma \vdash t_2 : \tau \quad \Gamma \vdash t_3 : \tau}{\Gamma \vdash \text{if } t_1 \text{ then } t_2 \text{ else } t_3 : \tau} \qquad \text{(T-If)}$$

$$\Gamma \vdash 0 : \text{Nat} \qquad \text{(T-Zero)}$$

$$\frac{t_1 : \text{N}at}{\Gamma \vdash \text{iszero } t_1 : \text{Bool}} \qquad \text{(T-IsZero)}$$

$$\frac{t_1 : \text{N}at}{\Gamma \vdash \text{succ } t_1 : \text{Nat}} \qquad \text{(T-Succ)}$$

$$\frac{t_1 : \text{N}at}{\Gamma \vdash \text{pred } t_1 : \text{Nat}} \qquad \text{(T-Pred)}$$

$$\frac{\text{x} : \tau \in \Gamma}{\Gamma \vdash \text{x} : \tau} \qquad \text{(T-Var)}$$

$$\frac{\Gamma, \text{x} : \alpha \vdash t_1 : \beta}{\Gamma \vdash \lambda \text{x} : \alpha.\ t_1 : \alpha \to \beta} \qquad \text{(T-Abs)}$$

$$\frac{\Gamma \vdash t_1 : \alpha \to \beta \quad \Gamma \vdash t_2 : \alpha}{\Gamma \vdash t_1\ t_2 : \beta} \qquad \text{(T-App)}$$