

## Ungraded Problem Set #4

1. Using the typing rules presented on the last two pages (including the new, combined S-RCD rule), give a type derivation for each of the following assuming the specified type context ( $\Gamma$ ) to start. If a type cannot be found for a term, clearly indicate where the derivation fails (show the derivation to that point).

- $(\lambda r : \{x:\text{Int}\}. r) \{x = 2, y = 3\}$   
where  $\Gamma = \{\}$
- $(\lambda r : \{x:\text{Int}, y:\text{Int}, z:\text{Int}\}. r) a$   
where  $\Gamma = \{a : \{x:\text{Int}, y:\text{Int}\}\}$
- $f g$   
where  $\Gamma = \{$   
 $f : (\{y:\text{Int}, x:\text{Int}\} \rightarrow (\{a:\text{Int}, b:\text{Int}\} \rightarrow \{m:\text{Int}\})) \rightarrow \text{Int},$   
 $g : \{y:\text{Int}\} \rightarrow (\{a:\text{Int}\} \rightarrow \{n:\text{Int}, m:\text{Int}\})$   
 $\}$

(note: You will likely want to use aliases for each of the record types if you write this out.)

2. For each of the following function applications, give the name of the record subtyping rules (S-RCDWIDTH, S-RCDPERM, S-RCDDEPTH, but not using the combined S-RCD) that apply (do not list permutation in the case that the field order remains unchanged).

- $f r$   
where  $\Gamma = \{$   
 $f : \{x:\text{Int}, y:\text{Int}\} \rightarrow \text{Int},$   
 $r : \{y:\text{Int}, x:\text{Int}\}$   
 $\}$
- $f r$   
where  $\Gamma = \{$   
 $f : \{x:\text{Int}\} \rightarrow \text{Int},$   
 $r : \{y:\text{Int}, x:\text{Int}\}$   
 $\}$
- $f r$   
where  $\Gamma = \{$   
 $f : \{x:\{n:\text{Int}\}\} \rightarrow \text{Int},$   
 $r : \{x:\{n:\text{Int}, m:\text{Int}\}, y:\text{Int}\}$   
 $\}$
- $f r$   
where  $\Gamma = \{$   
 $f : \{x:\{n:\text{Int}\} \rightarrow \{x:\text{Int}\}\} \rightarrow \text{Int},$   
 $r : \{x:\{n:\text{Int}\} \rightarrow \{x:\text{Int}\}\}$   
 $\}$
- $f r$   
where  $\Gamma = \{$   
 $f : \{x:\{n:\text{Int}\} \rightarrow \{x:\text{Int}\}\} \rightarrow \text{Int},$   
 $r : \{x:\{n:\text{Int}\} \rightarrow \{x:\text{Int}, y:\text{Int}\}\}$   
 $\}$

3. For each of the applications in the previous problem, clearly indicate whether or not the application is guaranteed to preserve typing if assignment (mutation) were supported by the language (considering that it is not clear what might happen within  $f$ ).

4. Download the provided Java source files. Read through the code and compile it to verify that there are no type errors. Experiment with changes to the parameter and return types for the functions `toBeCalled` and `actualFunction`. Be sure to try the incorrect variance for the parameter and for the return type; explain the errors in terms of subtyping on functions.

The terms for the expression language are to be inferred from the rules below coupled with the discussion in lecture (and in the textbook).

$nv$  is for numeric values.

$$\Gamma \vdash \text{true} : \text{Bool} \quad (\text{T-TRUE})$$

$$\Gamma \vdash \text{false} : \text{Bool} \quad (\text{T-FALSE})$$

$$\frac{\Gamma \vdash t_1 : \text{Bool} \quad \Gamma \vdash t_2 : \tau \quad \Gamma \vdash t_3 : \tau}{\Gamma \vdash \text{if } t_1 \text{ then } t_2 \text{ else } t_3 : \tau} \quad (\text{T-IF})$$

$$\Gamma \vdash nv : \text{Int} \quad (\text{T-INTCONST})$$

$$\frac{t_1 : \text{Int}}{\Gamma \vdash \text{iszero } t_1 : \text{Bool}} \quad (\text{T-ISZERO})$$

$$\frac{\Gamma \vdash t_1 : \text{Int} \quad \Gamma \vdash t_2 : \text{Int}}{\Gamma \vdash t_1 + t_2 : \text{Int}} \quad (\text{T-ADD})$$

$$\frac{\Gamma \vdash t_1 : \text{Int} \quad \Gamma \vdash t_2 : \text{Int}}{\Gamma \vdash t_1 - t_2 : \text{Int}} \quad (\text{T-SUB})$$

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \quad (\text{T-VAR})$$

$$\frac{\Gamma, x : \alpha \vdash t_1 : \beta}{\Gamma \vdash \lambda x. \alpha. t_1 : \alpha \rightarrow \beta} \quad (\text{T-ABS})$$

$$\frac{\Gamma \vdash t_1 : \alpha \rightarrow \beta \quad \Gamma \vdash t_2 : \alpha}{\Gamma \vdash t_1 t_2 : \beta} \quad (\text{T-APP})$$

$$\frac{\text{for each } i \quad \Gamma \vdash t_i : \tau_i}{\Gamma \vdash \{l_i = t_i^{i \in 1..n}\} : \{l_i : \tau_i^{i \in 1..n}\}} \quad (\text{T-RCD})$$

$$\frac{\Gamma \vdash t_1 : \{l_i : \tau_i^{i \in 1..n}\}}{\Gamma \vdash t_1.j : \tau_j} \quad (\text{T-PROJ})$$

$$\frac{\Gamma \vdash t_j : \tau_j}{\Gamma \vdash \langle l_j = t_j \rangle \text{ as } \langle l_i : \tau_i^{i \in 1..n} \rangle : \langle l_i : \tau_i^{i \in 1..n} \rangle} \quad (\text{T-VARIANT})$$

$$\frac{\Gamma \vdash t_0 : \langle l_i : \tau_i^{i \in 1..n} \rangle \quad \text{for each } i \quad \Gamma, x_i : \tau_i \vdash t_i : \tau}{\Gamma \vdash \text{case } t_0 \text{ of } \langle l_i = x_i \rangle \Rightarrow t_i^{i \in 1..n} : \tau} \quad (\text{T-CASE})$$

$$\Gamma \vdash \text{unit} : \text{Unit} \quad (\text{T-UNIT})$$

$$S <: S \quad (\text{S-REFL})$$

$$\frac{S <: U \quad U <: T}{S <: T} \quad (\text{S-TRANS})$$

$$\frac{\Gamma \vdash t : S \quad S <: T}{\Gamma \vdash t : T} \quad (\text{S-SUB})$$

$$\frac{P_2 <: P_1 \quad R_1 <: R_2}{P_1 \rightarrow R_1 <: P_2 \rightarrow R_2} \quad (\text{S-ARROW})$$

$$\frac{\{l_i \mid i \in 1..n\} \subseteq \{k_j \mid j \in 1..m\} \quad k_j = l_i \text{ implies } \alpha_j <: \beta_i}{\{k_j : \alpha_j \mid j \in 1..m\} <: \{l_i : \beta_i \mid i \in 1..n\}} \quad (\text{S-RCD})$$