Homework 1.

Due date: Thursday, May 6, at the beginning of the class.

Note: The primary goal of the homework is to give you an idea of the types of questions that will be asked on the midterm exam.

Problem 1 *Exercise 3.1-1. (page 52, textbook).*

Hint: look up the definition of Θ, and apply it directly.

Problem 2 *Problem 3-2. (page 61, textbook).*

Problem 3 *Problem 3-3.a (pages 61–62, textbook).*

Hint: It is almost like sorting (i.e., you can compare any two functions based on their asymptotic behavior: the Ω, Θ and O notation representing the $\leq$, $=$ and $\geq$ comparisons.

Note: Look up the definition of $\log^*$ in the textbook, if you are not sure.

Problem 4

In class, we have studied two selection algorithms: finding the largest and the second largest elements of an array. Using the same intuition as for the FindSecondMax() algorithm for finding the second largest element of an array (using tournaments), propose an algorithm findThirdLargest().

1. Describe the algorithm using pseudocode.
2. Analyze the number of comparisons the algorithm will use.

   Note: Your algorithm shall use (asymptotically) fewer comparisons than the **brute-force** algorithm of selecting the largest number in an array three times in a row.

Problem 5 *Exercise 16.2-1. (page 427, textbook)*

The Fractional Knapsack problem is defined in the textbook (see page 426) and in the lecture notes.

The greedy choice made by the optimal algorithm is to select the remaining item with the highest per-unit cost, and fill as much of the remaining knapsack space with it, as possible.

Problem 6 *Exercise 12.2-4. (pages 427–428, textbook)*

Spell out how the optimal substructure of this problem looks like.

Determine what a good greedy choice is for this problem and prove that it does, indeed, lead to an optimal solution.

Provide pseudocode for the algorithm.

Problem 7

Determine Huffman Codes for each of the following texts. You do not need to translate the text, but, please, show the tree. Encode only the English alphabet letters that are found in each text. All text is case-insensitive.

1. `A smallish simple test.`
2. `ATCTTGTCGTTGCA`
3. `ATGATCATGATCATT`
4. `A certain king had a beautiful garden, and in the garden stood a tree which bore golden apples.`
5. `HP and Palm Inc today announced that they have entered into a definitive agreement under which HP will purchase Palm a provider of smartphones powered by the Palm webOS mobile operating system.`
6. `Bellardo Buckalew Clark Clements Dekhtyar Fisher Gharibyan Haungs Janzen Keen Kurfess Lupo Nico Seng Smith Staley Turner Vakalis Wood`

Problem 8

Consider the following instance of the Rod Cutting problem:

$n = 9$;

length i	1	2	3	4	5	6	7	8	9
Price[i]	1	3	4	6	8	9	11	12	13

Find the optimal solution for this instance. Show all work. (i.e., I expect you to compute the optimal cost for each $1 \leq i \leq 9$.)

Problem 9

Consider the following two instances of Make Change problem:

$M1 = \{1, 3, 7, 18, 23\}$
$M2 = \{1, 4, 13, 21, 26\}$

Using both dynamic programming and greedy approaches, compute change for 100 for each of the two instances.

Show all work.