Functional Dependencies in Databases
Examples

Functional Dependencies

Example 1: Functional Dependencies

Consider the following relation Purchases:

Purchases(ReceiptNo,Ordinal, PDate, Customer, Item, Price)

Each tuple of the relation stores information about a single item purchased by a customer. The information stored is the receipt number, the position of the item on the receipt (item scanned first has Ordinal = 1, next item — Ordinal = 2, etc.), the date of the purchase, the name of the customer, the name of the purchased item and its price. Consider the following fragment of the Purchases table.

ReceiptNo	Ordinal	PDate	Customer	Item	Price
1234	1	01/21/2008	"Fausto Klosner"	"Chocolate Chip Cookie"	0.95
1234	2	01/21/2008	"Fausto Klosner"	"Rye Bread"	2.50
3042	1	01/23/2008	"Susie Gladney"	"Rye Bread"	2.50
3042	2	01/23/2008	"Susie Gladney"	"Opera Cake"	15.00
3403	1	01/26/2008	"Susie Gladney"	"Ganache Cookie"	1.30
5612	1	01/26/2008	"Dean Dews"	"Chocolate Chip Cookie"	0.95

We can make the following statements about the Purchases relation:

1. Receipt number uniquely identifies the customer.
2. Receipt number uniquely identifies the date of purchase.
3. Name of the item uniquely identifies the price of the item.
4. Receipt number and the ordinal (position on the receipt) uniquely identify the purchased item.

These translate into the following functional dependencies:

1. ReceiptNo $\rightarrow$ Customer
2. ReceiptNo $\rightarrow$ PDate

3. Item → Price

4. ReceiptNo, Ordinal → Item

Example 2: Keys and Superkeys

Example 2.1. Consider a relation $R(A, B, C, D)$ with the folloing set of FDs:

$A \rightarrow C$
$B \rightarrow D$

Question: Find a key of R.

Solution: A key is a minimal collection of attributes that functionally determines all attrbiutes in a relation.

- Step 1. Can A alone be a key? A functionally determines A and via the first rule, it functionally determines C. But it does not determine B or D. A is **not a key** of R.
- Step 2. Can B alone be a key? B functionally determines B and via the second rule, it functionally determines D. But it does not determine A or C. B is **not a key** of R.
- Step 3. Can C alone be a key? C functionally determines C. It does not determine A, B or D. C is **not a key** of R.
- Step 4. Can D alone be a key? D functionally determines D. It does not determine A, B or C. D is **not a key** of R.
- Step 5. Can A, B be a key? A functionally determines A and via the first rule, it functionally determines C. B functionally determines B and via the second rule, it functionally determines D. Thus, A, B, functionally determine A, B, C and D - **all attributes** of R. Since neither A nor B is a key by itself (Steps 1 and 2), A, B **is a key**.

Example 2.2. Consider a relation $R(A, B, C, D, E)$ with the following set of FDs:

$A, B \rightarrow C$
$B \rightarrow D$
$C \rightarrow D$

Question(s): How many keys does R have? Can you name them?

Solution:

Example 3: FD derivations

Example 3.1. Consider a relation $R(A, B, C, D, E, F)$ with the following set of FDs:

$A, B \to C, F$
$C \to D$
$B, D \to C$

Question: Find a key for R.

Solution: We use the split rule to replace $A, B \to C, F$ with two FDs:

$A, B \to C$
$A, B \to F$

It is clear that no singleton (i.e., A only, B only, C only, D only, E only, F only) is a key.

Consider now, a pair A, B of attributes. Trivial dependency (reflexivity) rules $A, B \to A$ and $A, B \to B$ hold. Therefore, combining with the two split FDs from above, we obtain: A, B functionally determines attributes A, B, C, F.

Now, we consider the FDs $A, B \to C$ and $C \to D$. By transitivity rule, we derive a new FD:

$$A, B \to D$$

Thus, A, B functionally determine A, B, C, D, F. There is one more attribute left — E. This attribute is not part of any functional dependencies. Thus, A, B is not a key.

We note now, that A, B, E functionally determine all attributes of R: A, B, C, D, E, F, hence A, B, E is a **superkey**. Since niether B, E nor A, E is a key (check yourselves), A, B, E is a **key** of R.

Example 3.2. Consider a relation $R(A, B, C, D, E, F)$ with the following set of FDs:

$A \to B$
$F, C \to D, A$
$C \to B$
$A, E \to C$

Question: Find a key of R.

Solution:

Example 3.3. Find a key for relation Purchases from Example 1.

Example 4: Closure

Example 4.1. Consider a relation $R(A, B, C, D, E, F)$ with the following set of FDs:

$A \to B$
$B, F \to D, E$
$C \to B$
$A, E \to C$

Question: Find $\{A, F\}^+$.

Solution: We use the FD-Closure algorithm.

Step 0. We split $B, F \to D, E$ into $B, F \to D$ and $B, F \to E$.

Step 1. $X := \{A, F\}$.

Step 2. Left side of FD $A \to B$ is in X. $X := X \cup \{B\} = \{A, F, B\}$.

Step 3. Left side of FD $B, F \to D$ is in X. $X := X \cup \{D\} = \{A, F, B, D\}$.

Step 3. Left side of FD $B, F \to E$ is in X. $X := X \cup \{E\} = \{A, F, B, D, E\}$.

Step 4. Left side of FD $A, E \to C$ is in X. $X := X \cup \{C\} = \{A, F, B, D, E, C\}$

Step 5. X contains all attrbitues of R. Therefore $\{A, F\}^+ = X = \{A, B, C, D, E, F\}$.

Example 4.2. Consider a relation $R(A, B, C, D, E, F)$ with the following set of FDs:

$A \to C, E$
$A, B \to D$
$C, F \to B$
$F, E \to D$

Question: Find $\{A, B\}^+$.

Solution:

Question: Find $\{F, E, C\}^+$.

Solution: