Data 401
Nonlinear Regression and Classification

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October 31, 2016

1. Review
2. k-Nearest Neighbors
3. Decision Trees

Linear Methods

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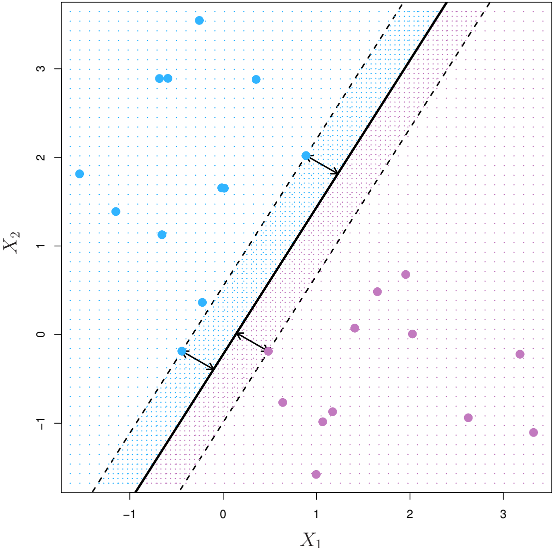
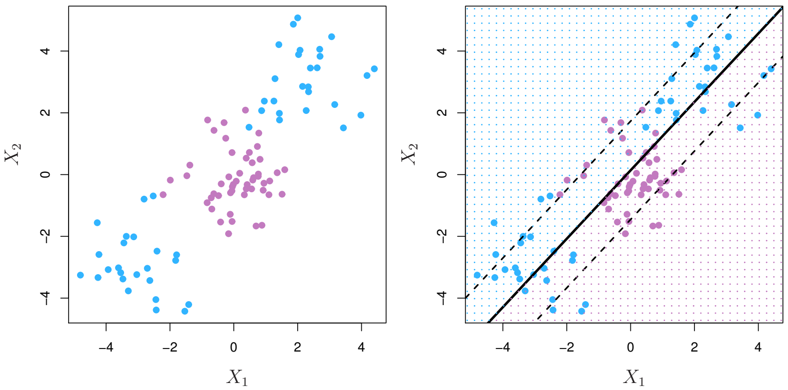
  - **Perceptron**: Fit linear regression to binary data, classify by thresholding predicted values.
  - **Support vector machines** (SVM)

Linear Classifiers

The linear classifiers are linear because their decision boundaries are linear.



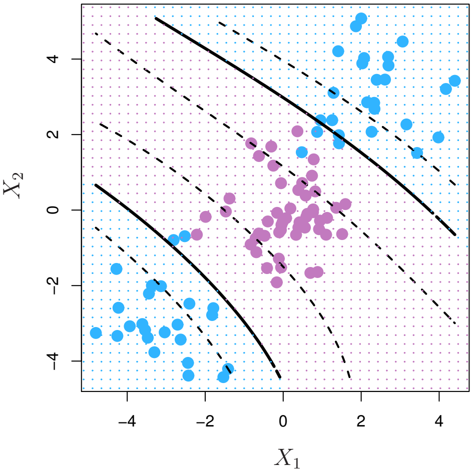
The Need for Nonlinear Classifiers



Nonlinear Methods

One way to add nonlinearity is through **basis expansions**:

$$X_1 \mapsto (X_1, X_1^2)$$
$$X_2 \mapsto (X_2, X_2^2)$$



Nonlinear Methods

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In this lecture, we'll learn about two methods that are fundamentally non-linear: k**-nearest neighbors** and **decision trees**.

I want you to think about what the regression functions and the decision boundaries look like for these methods.

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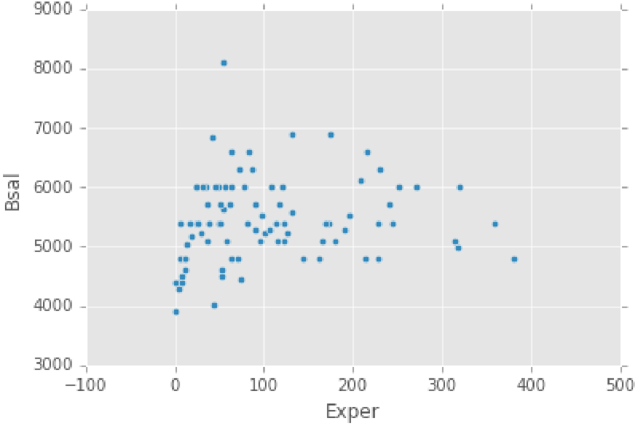
Intuition: These k neighbors are most similar to $\mathbf{x}^*$, so they should share a similar response.

k-Nearest Neighbors Regression

This data is taken from the Harris bank data set. The data can be found on JupyterHub at `/data/harris.csv`.



k-Nearest Neighbors Regression in Scikit-Learn

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from sklearn.neighbors import KNeighborsRegressor
model = KNeighborsRegressor(n_neighbors=5, weights="uniform")
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Let's investigate what the regression function estimated by k-nearest neighbors looks like.

In-Class Exercise

- *Add the curve that is fitted by k-nearest neighbors to the plot below. What happens as you vary k?*
- *Use cross-validation to select the optimal k.*

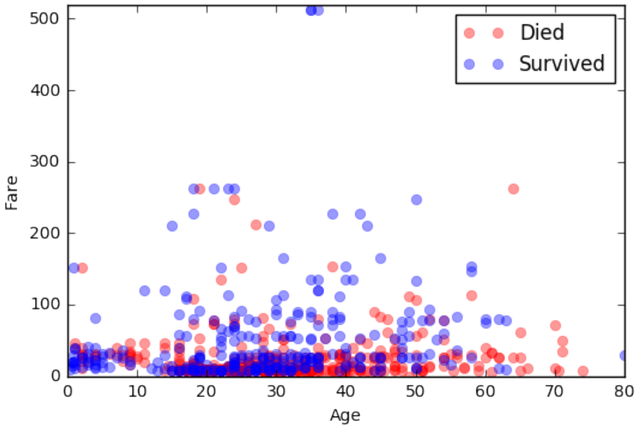
k-Nearest Neighbors Classification

The k-nearest neighbors classifier predicts the output of a test input by *majority vote* based on the outputs of the k “closest” points in the training data.

k-Nearest Neighbors Classification in Pictures

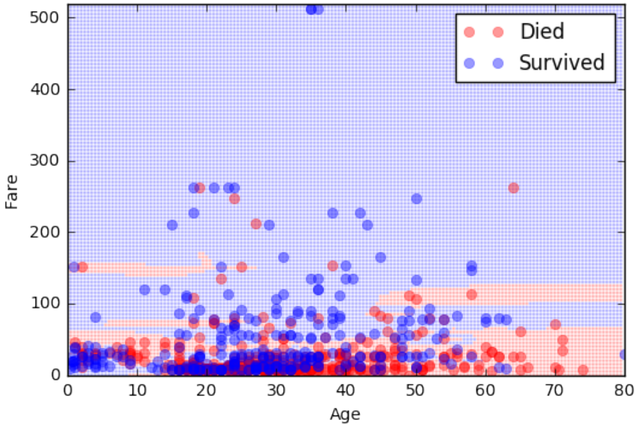
This data is taken from the Titanic data set. The data can be found on JupyterHub at `/data/titanic.csv`.

Let's just consider two variables, `Age` and `Fare`.



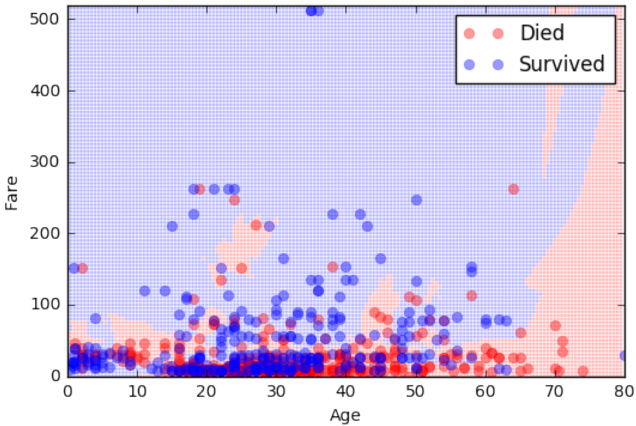
k-Nearest Neighbors Classification in Pictures

Here are the predictions returned by 5-nearest neighbors:



k-Nearest Neighbors Classification in Pictures

Notice that `Fare` and `Age` are on different scales. So I transformed `Fare` to $10\log(1+\texttt{Fare})$ before calculating distances. And the predictions I get are very different!



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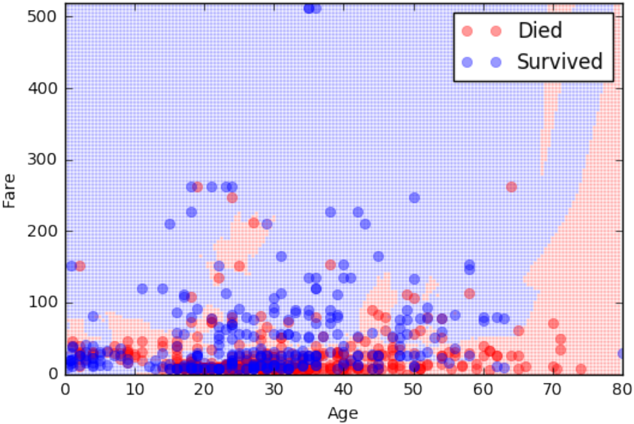

K-nearest neighbors is sensitive to the choice of distance metric d.

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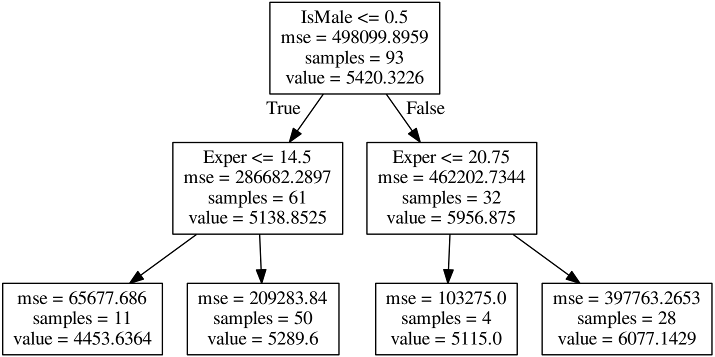
Decision Trees

Decision tree for predicting starting salary, trained on the Harris bank data.



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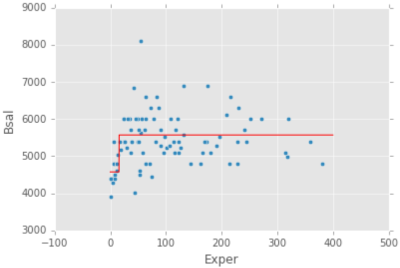

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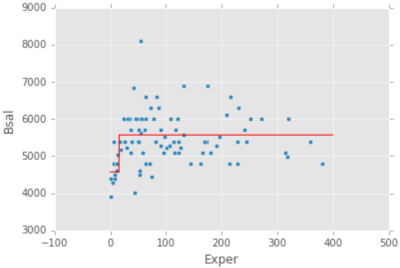

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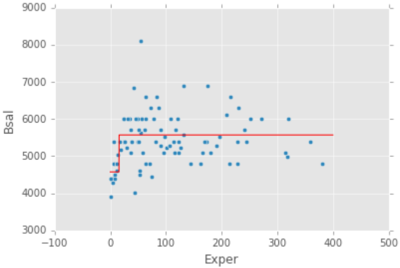

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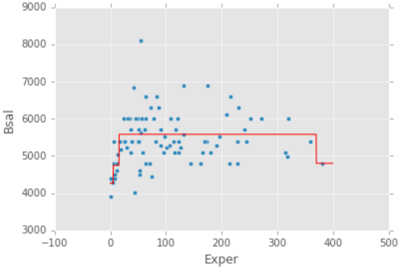

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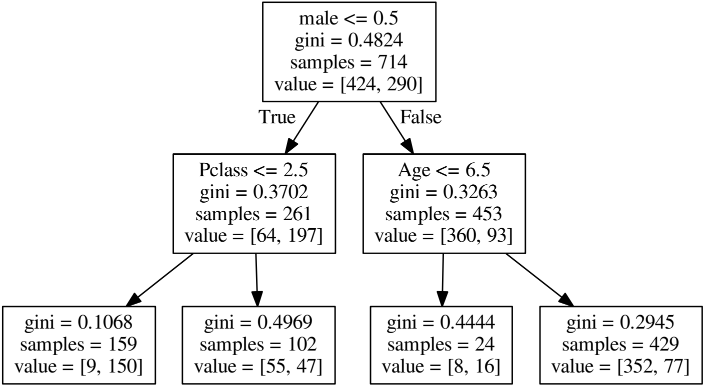
In-Class Exercise

- *Fit a decision tree to the Harris bank data to predict beginning salary (`Bsal`) from experience (`Exper`). Plot the estimated regression function. What happens as you vary the tree depth?*
- *Use cross-validation to select the optimal tree depth.*

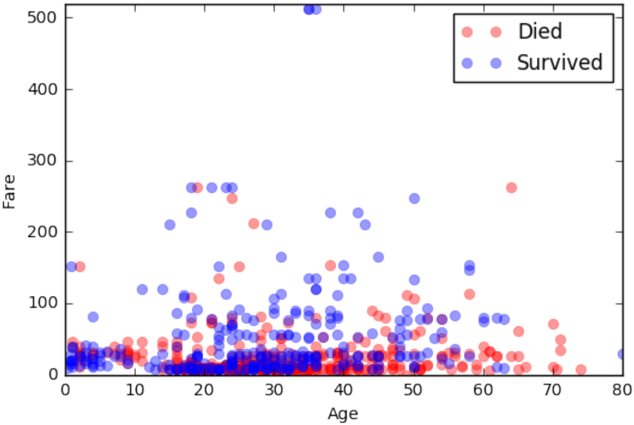
Decision Tree Classification

Decision tree for predicting survival on the Titanic.



Decision Tree Classification in Pictures

Let's again just consider two variables, `Age` and `Fare`. What will the decision boundary of a decision tree classifier look like?



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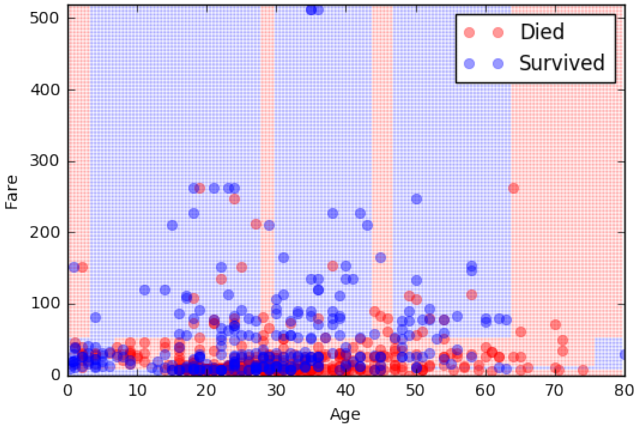

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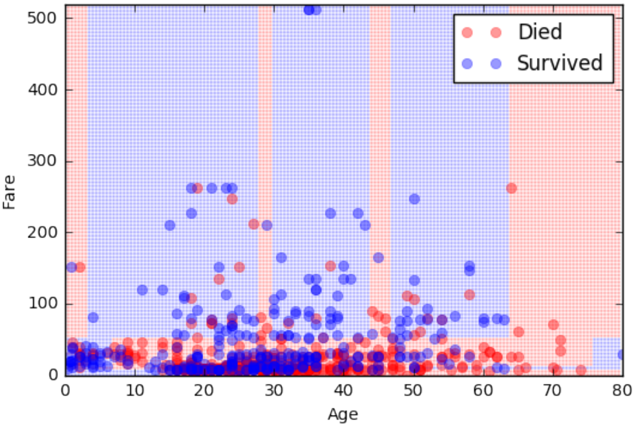

When you fit a decision tree, the predicted value tends to be constant over rectangular regions. This is because decision trees only split on one variable at a time.

Decision Tree Classification in Scikit-Learn

```
from sklearn.tree import DecisionTreeClassifier
model = DecisionTreeClassifier(max_depth=5)
model.fit(X, y)
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