Methods For Working With Time Series: Hidden Markov Models & More

Hunter Glanz

California Polytechnic State University
San Luis Obispo

February 8, 2019

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Outline

1. Introduction to Time Series
2. Traditional Time Series Analysis
3. Introduction to Hidden Markov Models

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What is a *time series*?

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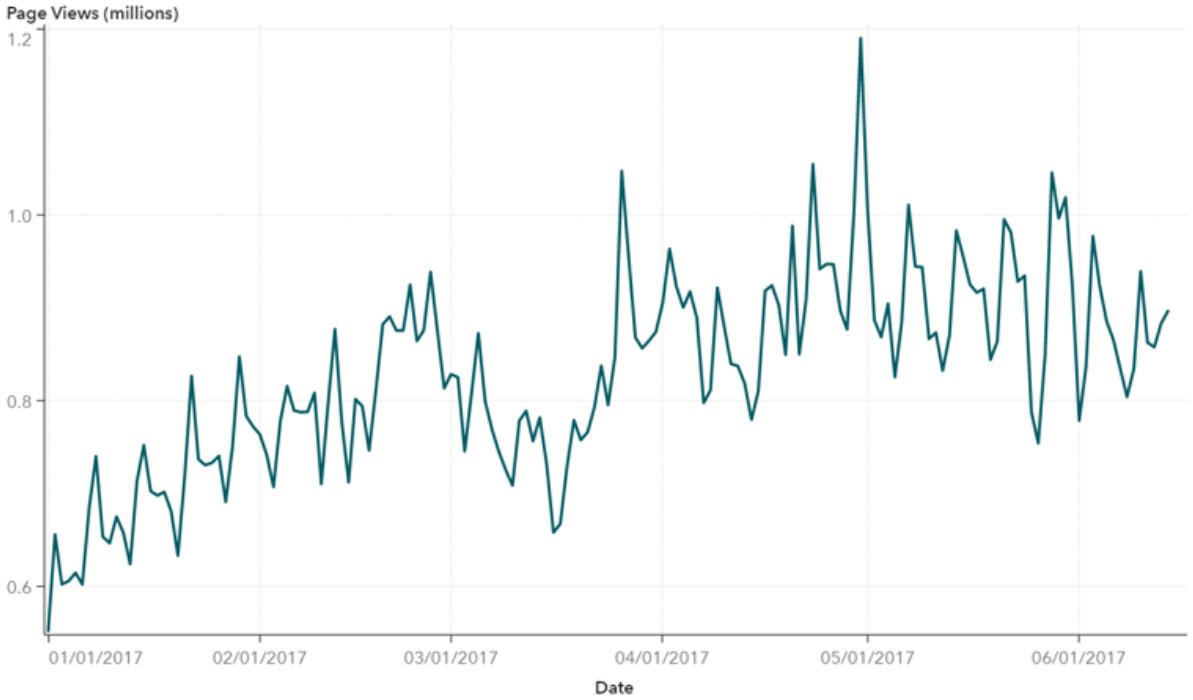

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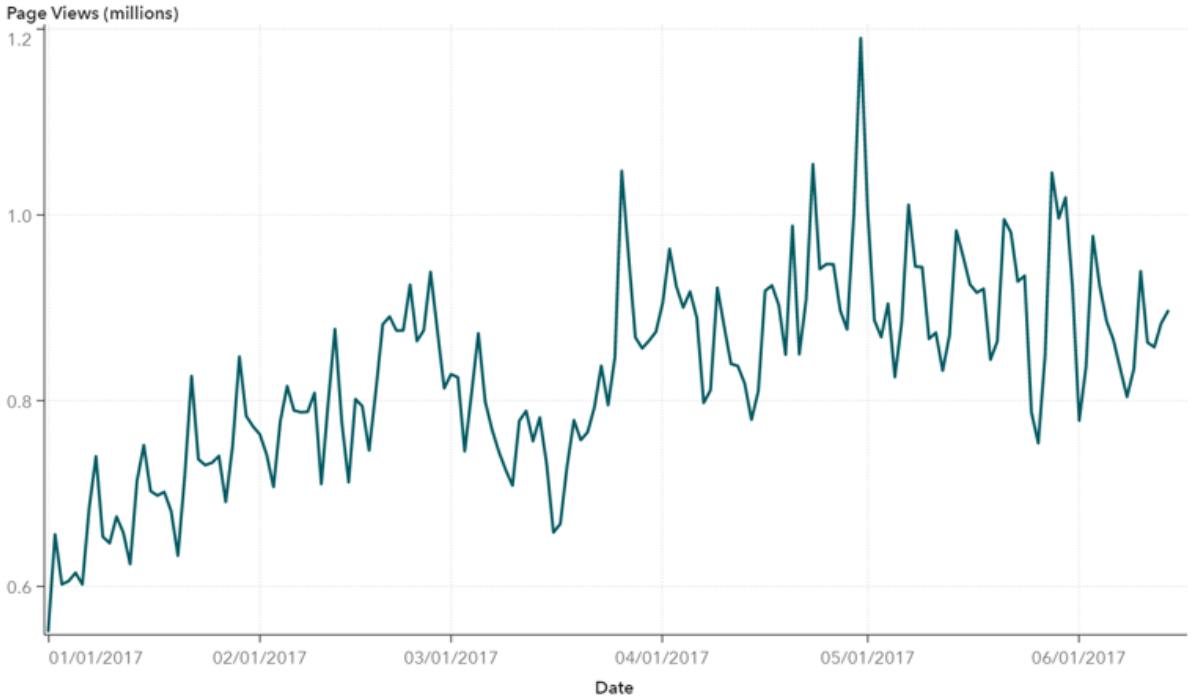

- **time series**: a series of data points indexed in time order; most commonly taken at successive equally spaced points in time.

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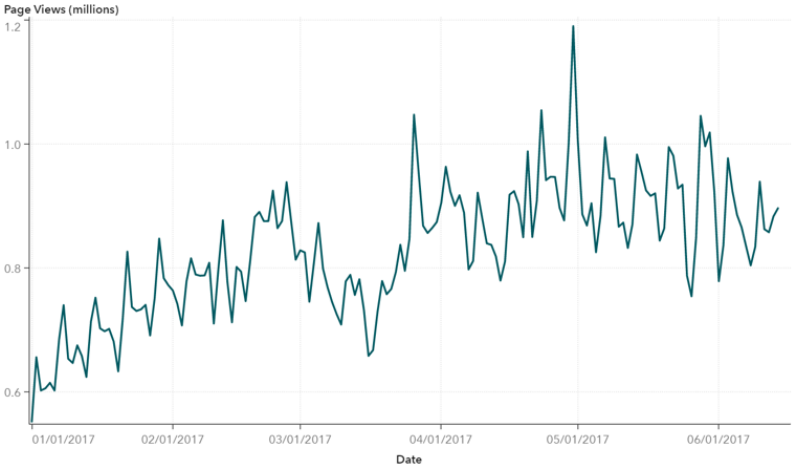
What Are We Interested In?



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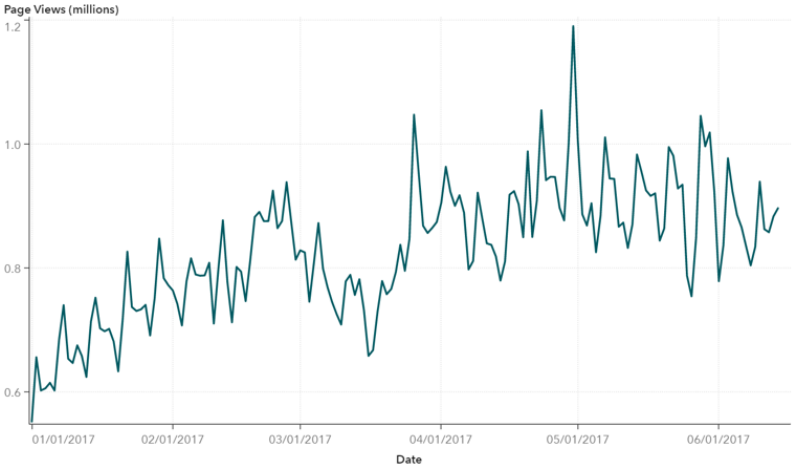

- Descriptive/Exploratory

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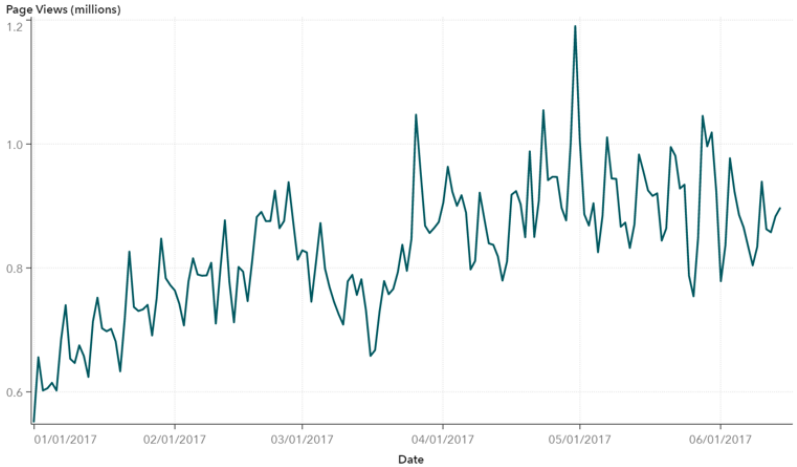

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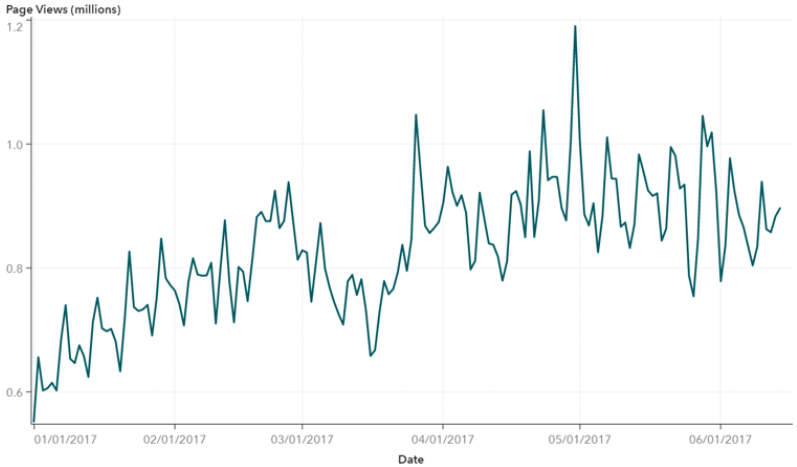

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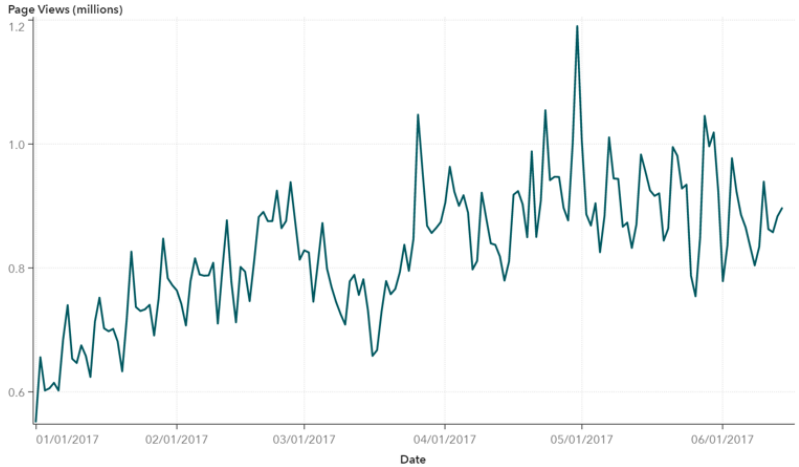

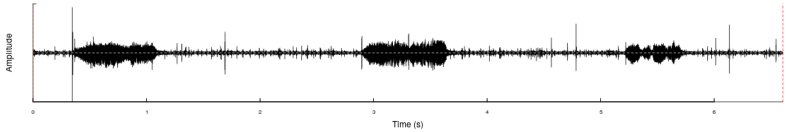

- Descriptive/Exploratory
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- **Segmentation/Classification**

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STAT 416: Statistical Analysis of Time Series

Analysis and forecasting of a single quantitative variable (time series)

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STAT 416: Statistical Analysis of Time Series

Analysis and forecasting of a single quantitative variable (time series)

- Autocorrelation
- Autoregressive (AR) models
- Moving Average (MA) models
- ARMA & ARIMA models

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Autocorrelation

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Correlation of a signal with a delayed copy of itself as a function of the delay

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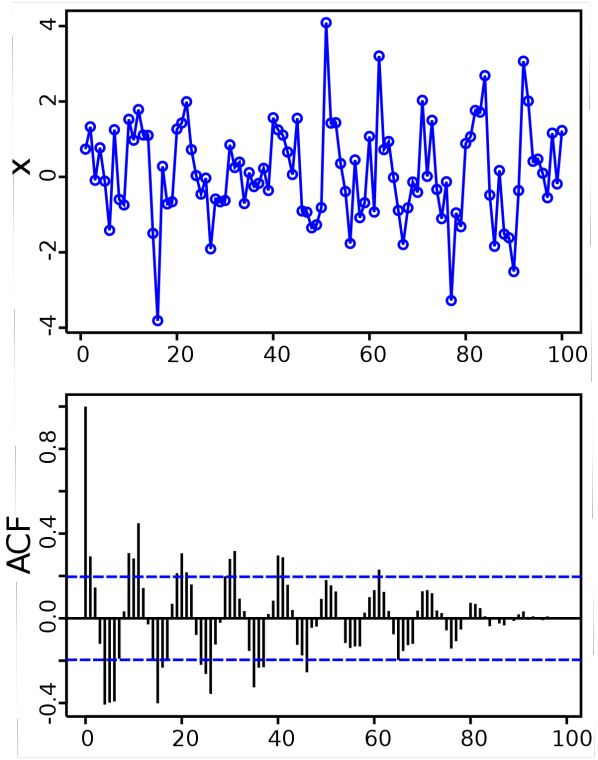

- **autocorrelation**: similarity between observations as a function of the time lag between them

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Autocorrelation

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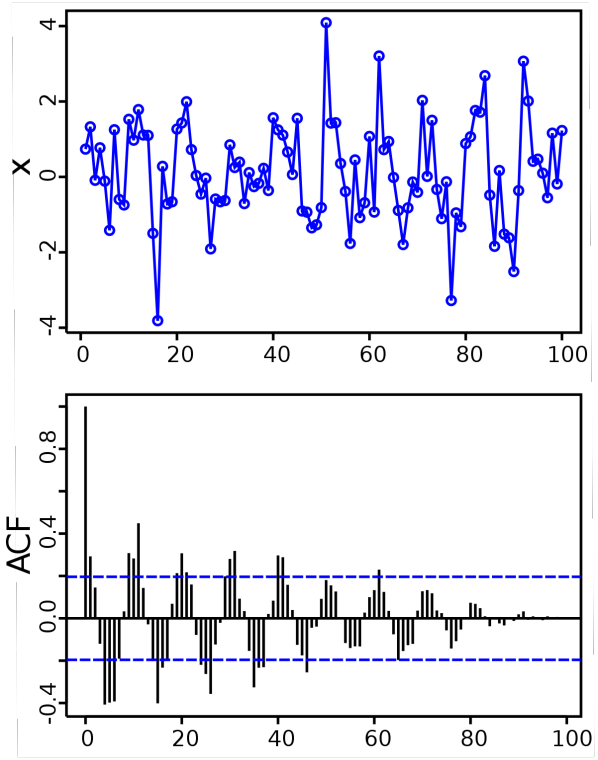

- **autocorrelation**: similarity between observations as a function of the time lag between them
- What could you conclude from the graph of the ACF?

Autoregressive Models

Autoregressive Models

- Autoregressive model of order p; $AR(p)$.

$$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t-i} + \varepsilon_t$$

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- How do we choose p?
- How do we estimate the φ coefficients?

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Moving Average Models

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- Moving Average model of order q; $MA(q)$.

$$X_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta \varepsilon_{t-q}$$

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ARMA and ARIMA Models

- $ARMA(p, q)$:

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$$X_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi X_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$$

- $ARIMA(p, d, q)$:
  - p is the order of the AR part of the model
  - q is the order of the MA part of the model
  - d is the degree of differencing of the data values

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And Beyond!

- Other methods:
  - spectral analysis
  - wavelet analysis
  - signal processing
  - statistical and machine learning methods

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And Beyond!

- Other methods:
  - spectral analysis
  - wavelet analysis
  - signal processing
  - statistical and machine learning methods
- Python Implementations
  - **Statsmodels**
  - PyFlux
  - PyMC3

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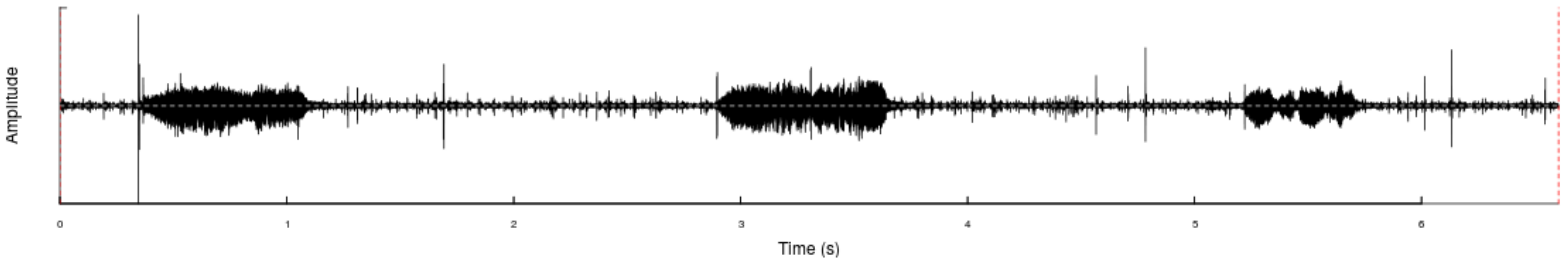
Segmentation and Classification



- What if we're not interested in forecasting a quantitative value?

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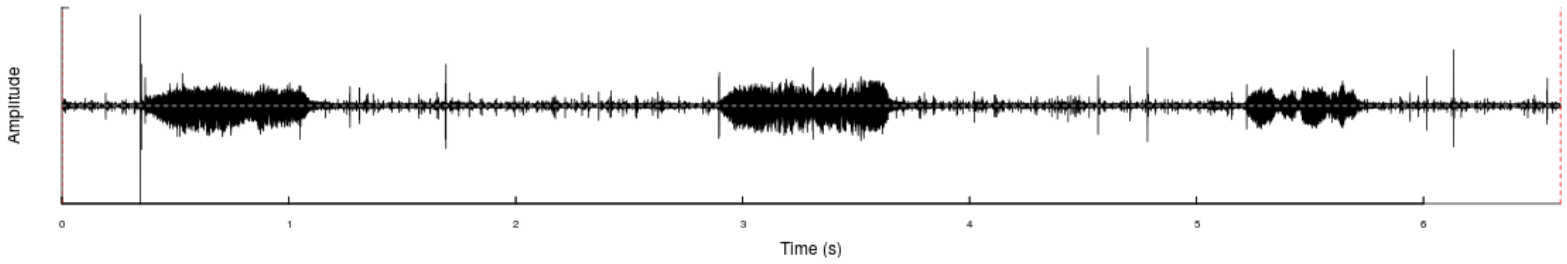

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- Segmentation/Change-point detection

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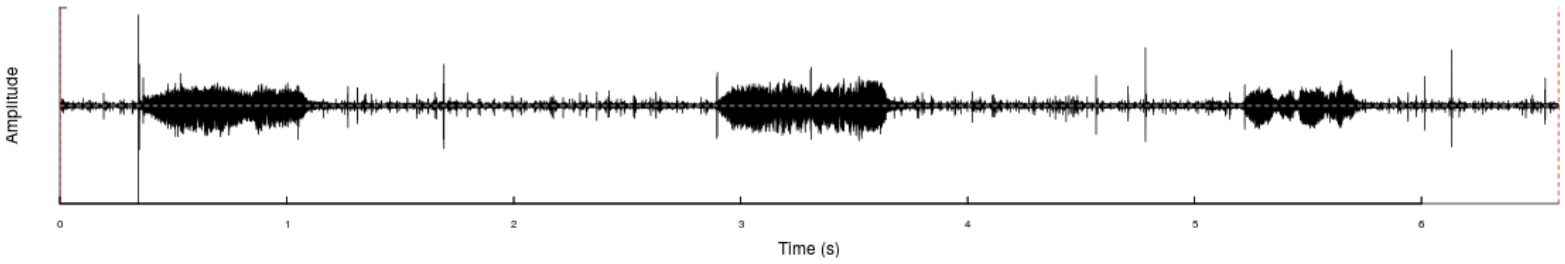
Segmentation and Classification



- What if we're not interested in forecasting a quantitative value?
- Segmentation/Change-point detection
- Segmentation $\Longrightarrow$ Classification

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Discrete Inference

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- **Hidden Markov Models (HMMs)**:
  - Fall under the umbrella of many different types of models
  - Well summarized by the following image:

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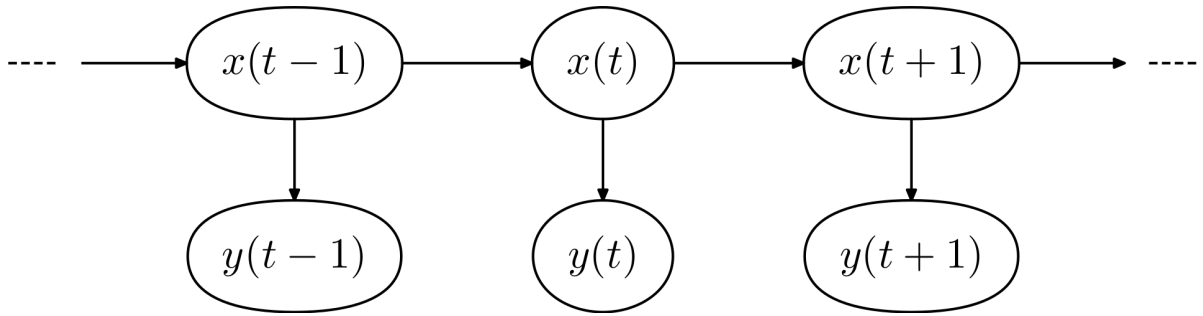

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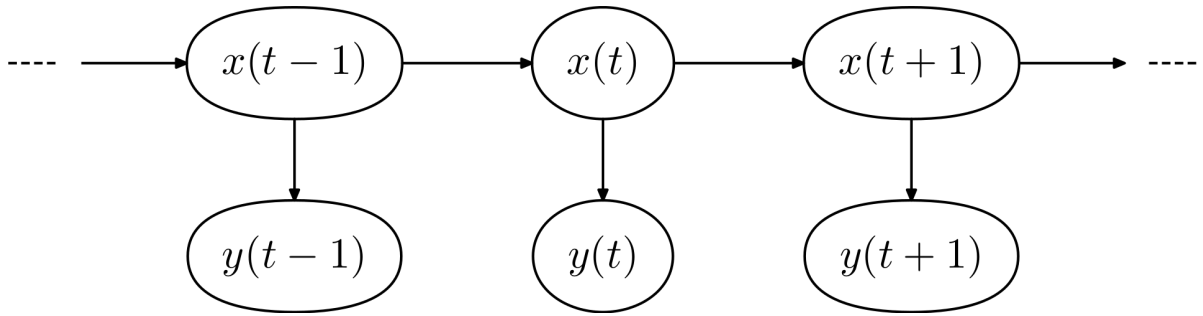
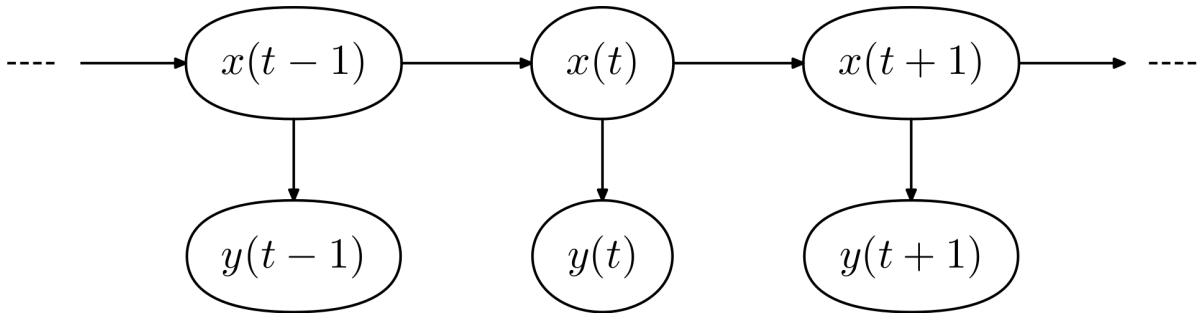


  - Dolphin/Whale calls; Keadle project

Properties of HMMs



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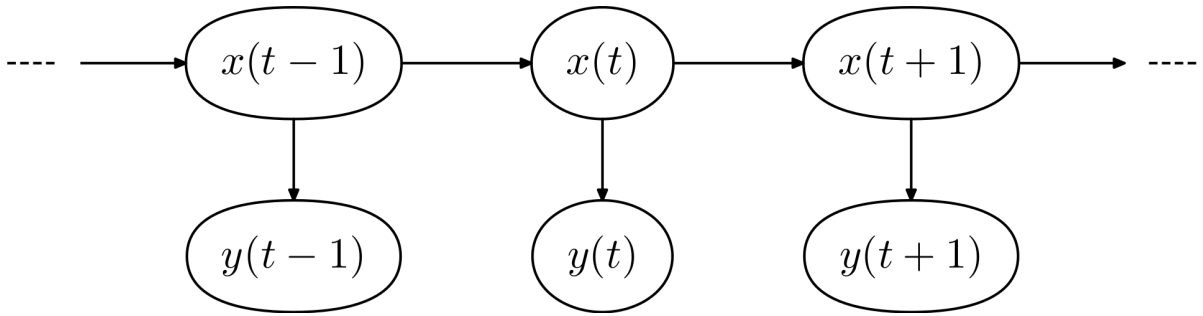

- **Markov property**: Conditional probability distribution of hidden variable, $x(t)$ at time t, depends <u>*only*</u> on the value of the hidden variable $x(t-1)$

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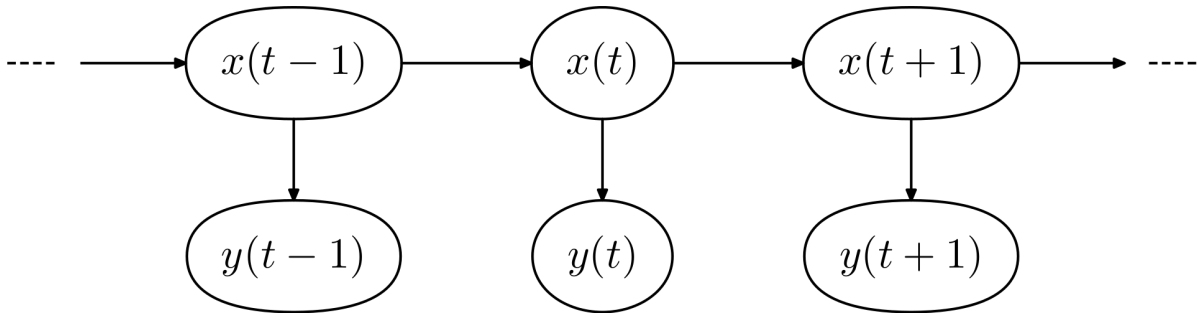

- **Markov property**: Conditional probability distribution of hidden variable, $x(t)$ at time t, depends <u>*only*</u> on the value of the hidden variable $x(t-1)$
- Value of the observed variable $y(t)$ depends <u>*only*</u> on the value of the hidden variable $x(t)$

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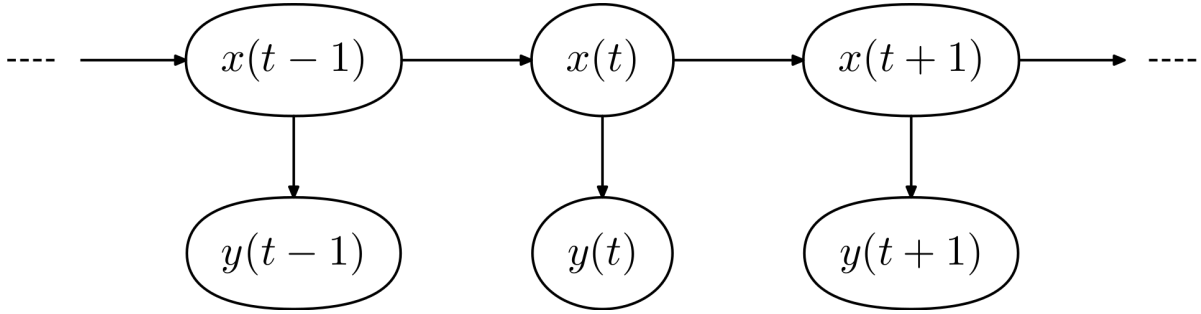
Characterizing HMMs



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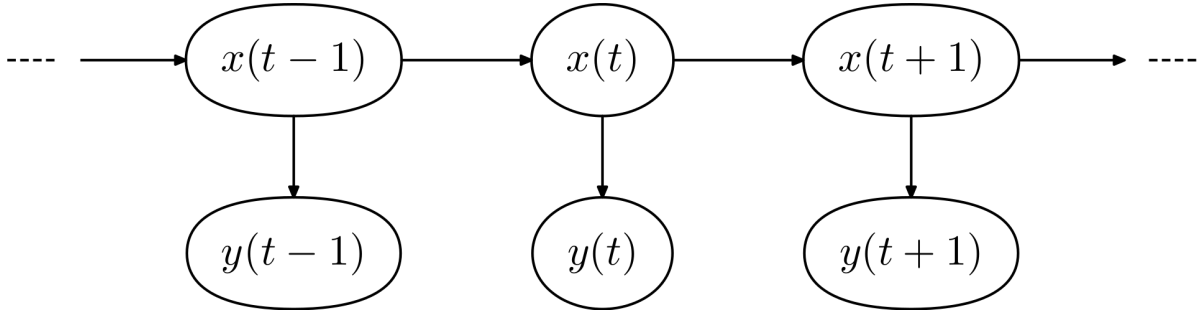
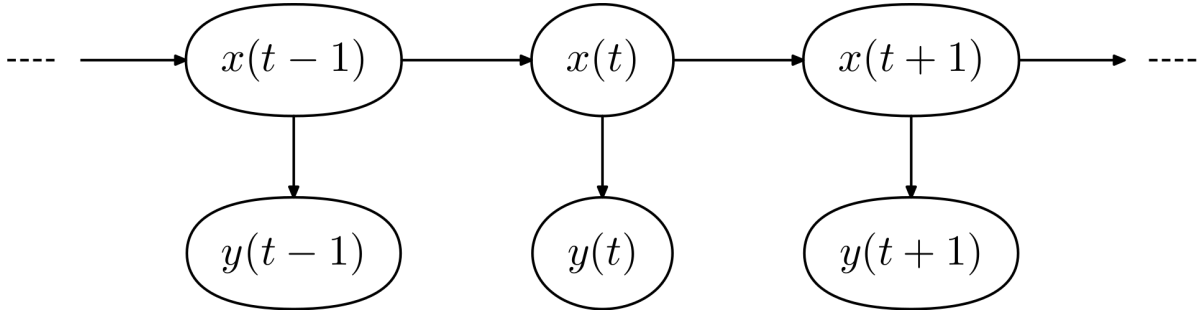


- Number of states

Characterizing HMMs



- Number of states
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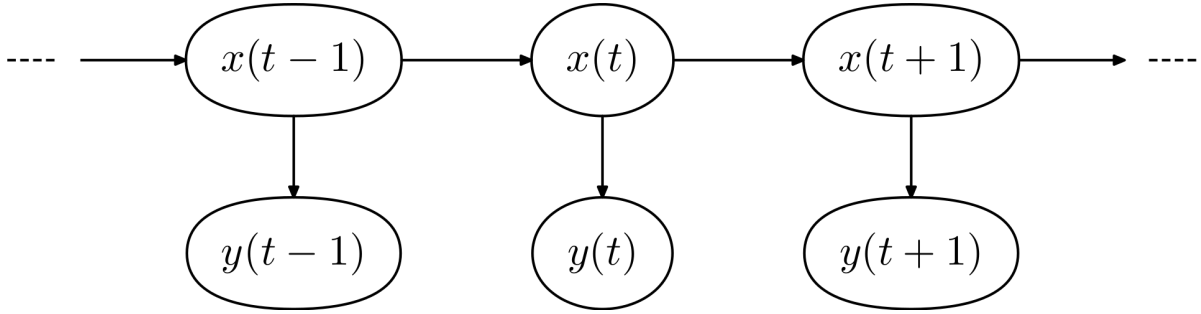
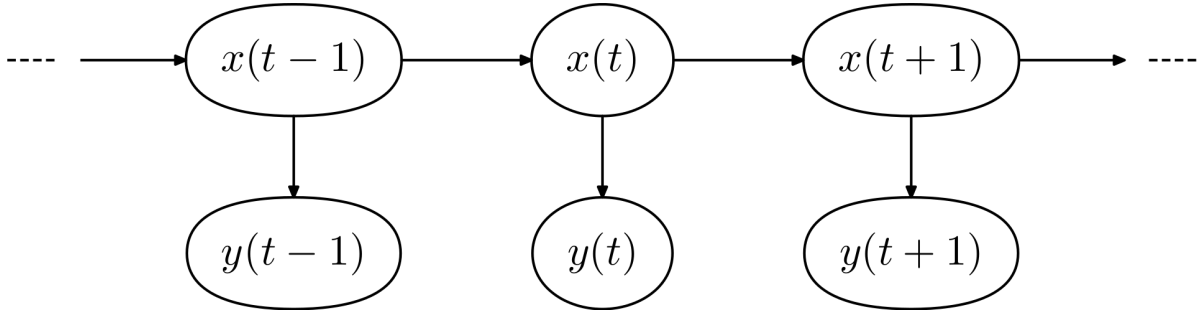


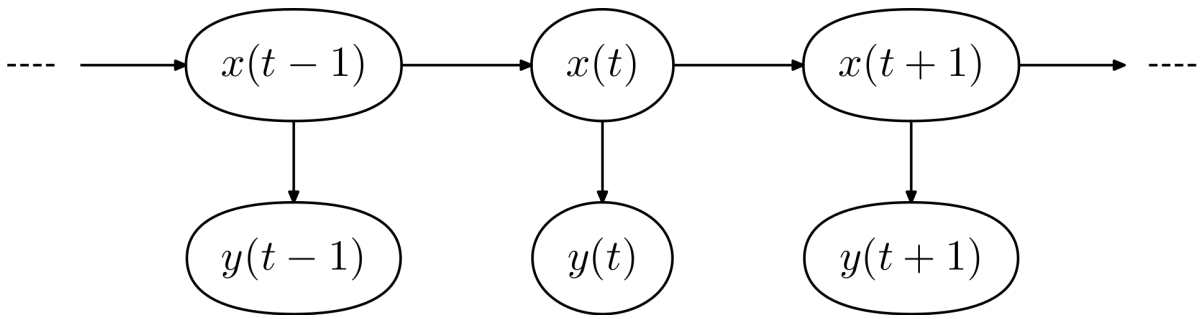
- Number of states
- Number of observations
- Emission probabilities

Characterizing HMMs



- Number of states
- Number of observations
- Emission probabilities
  - $P(Y_i|X_i)$

Characterizing HMMs



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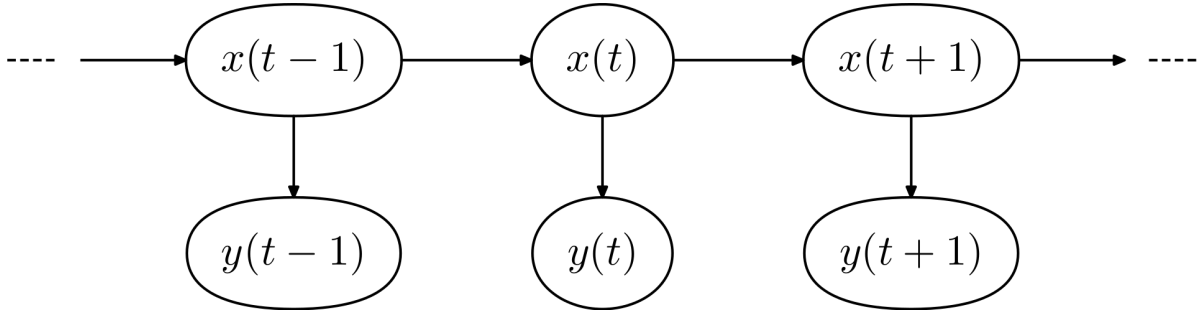
- Emission probabilities
  - $P(Y_i|X_i)$
- Transition probabilities

Characterizing HMMs



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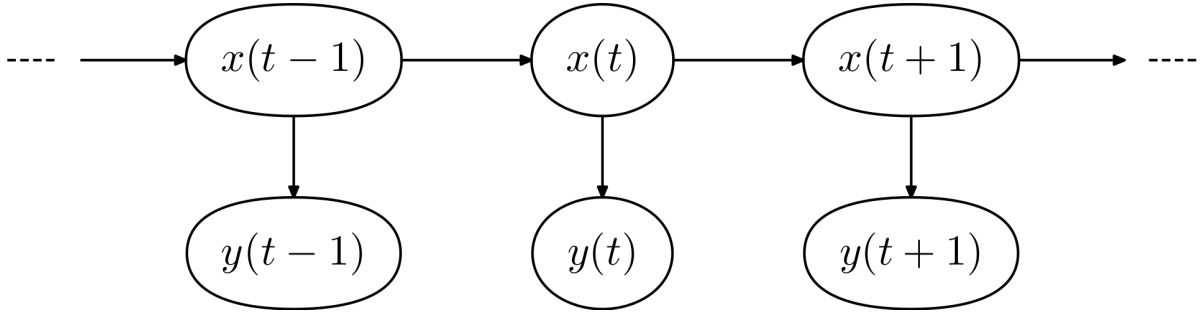

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- (Hidden) States

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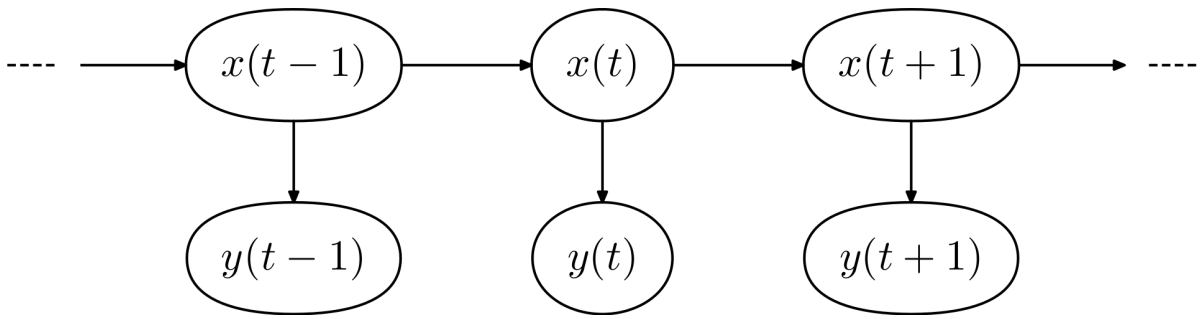

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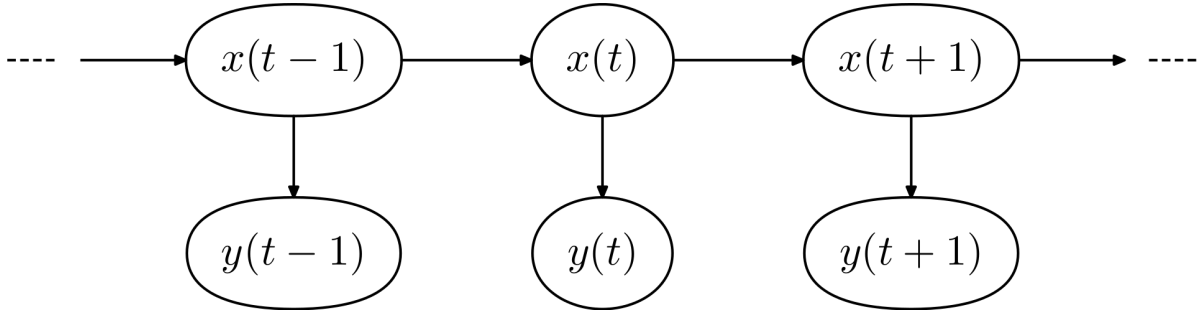

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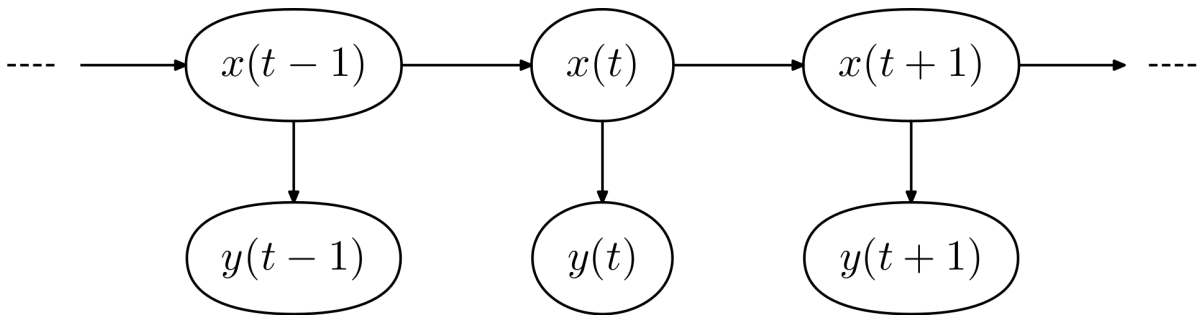

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- (Hidden) States
- Observations (data)
- Probability distribution(s)
- (Prior) Initial probabilities of states

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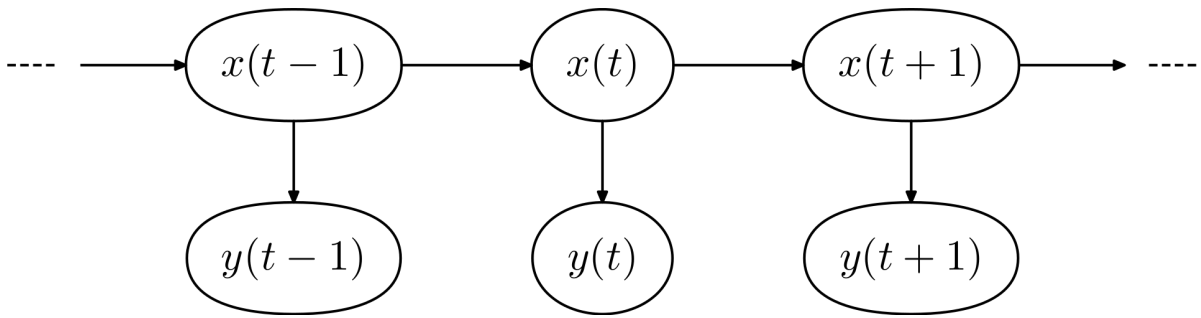

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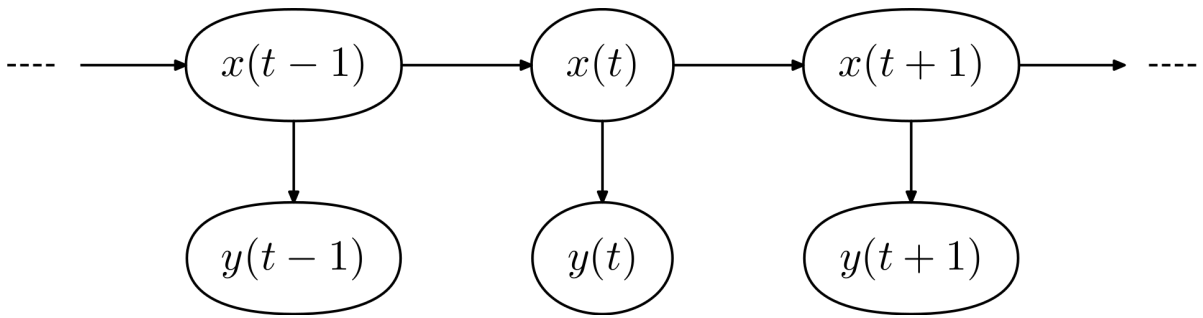

The States!!!

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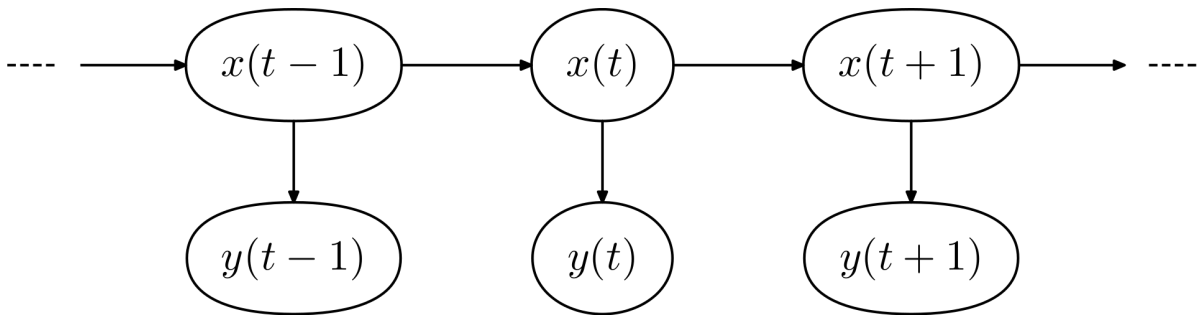

The States!!!

- How should we estimate the states?

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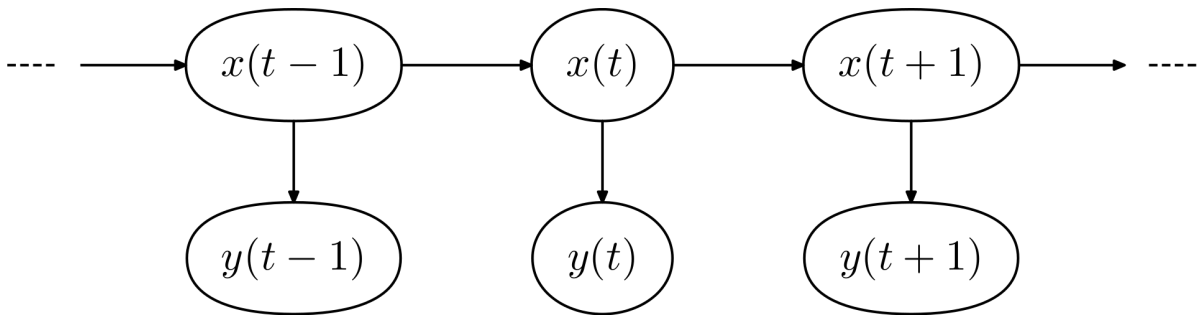

The States!!!

- How should we estimate the states?
- Python Implementations
  - hmmlearn
  - seqlearn

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HMM Inference

- Two things we might be interested in:

HMM Inference

- Two things we might be interested in:
  - Most likely sequence of hidden states (maximum a posteriori estimator)

$$\hat{X} = \underset{X}{\operatorname{argmax}}\, P(X|Y)$$

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HMM Inference

- Two things we might be interested in:
  - Most likely sequence of hidden states (maximum a posteriori estimator)

$$\hat{X} = \underset{X}{\operatorname{argmax}}\, P(X|Y)$$

  - Centroid estimator (unconstrained)

$$\tilde{X}_i = \underset{X_i \in S}{\operatorname{argmax}}\, P(X_i|Y)$$

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Maximum a Posteriori Estimator

- Small example…
- Viterbi Algorithm!

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Centroid Estimator

- Continuing our small example….

Questions?

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