1 Handling of Continuous Attributes in C4.5. Algorithm

**Notation.** Let \( D \) be a dataset over the list of attributes \( A = \{A_1, \ldots, A_n\} \). Let \( A_i \in A \) be a **continuous attribute**.

A **binary split** of \( D \) on attribute \( A_i \) at value \( \alpha \) is a pair \( D^- \subseteq D, D^+ \subseteq D \), such that:

1. \( D^- \cup D^+ = D \)
2. \( D^- \cap D^+ = \emptyset \)
3. \((\forall d \in D^-) d[A_i] \leq \alpha \)
4. \((\forall d \in D^+) d[A_i] > \alpha \)

**Idea.** On each step of **C4.5 Algorithm**, for each continuous attribute \( A_i \) find a **binary split** with the best information gain (or information gain ratio). More specifically, the entropy of a binary split of \( D \) on \( A_i \) using \( \alpha \) is

\[
\text{entropy}_{A_i, \alpha}(D) = -\frac{|D^-|}{|D|} \cdot \text{entropy}(D^-) - \frac{|D^+|}{|D|} \cdot \text{entropy}(D^+).
\]
function selectSplittingAttribute(A, D, threshold); // uses information gain
begin
  p0 := entropy(D);
  for each \( A_i \in A \) do
    if \( A_i \) is continuous then
      \( x := \text{findBestSplit}(A_i, D) \);
      \( p[A_i] := \text{entropy}_{A_i, x}(D) \);
    else
      \( p[A_i] := \text{entropy}_{A_i}(D) \);
    endif
    \( \text{Gain}[A_i] = p0 - p[A_i] \); // compute info gain
  endfor
  best := arg(\text{findMax}(\text{Gain})));
  if \( \text{Gain}[best] > \text{threshold} \) then return best
  else return NULL;
end

function findBestSplit(A_i, D) // finds best binary split for a continuous attribute
begin
  initialize associative arrays \( \text{counts}_1[], \ldots, \text{counts}_k[] \);
  initialize associative array \( \text{Gain} \);
  p0 := entropy(D);
  foreach \( d \in D \) do // Step 1: scan data
    for \( j = 1 \) to \( k \) do
      if \( \text{class}(d) == c_j \) then
        \( \text{counts}_j[d[A_i]] := \text{counts}_j[d[A_i]] + 1 \);
      else
        \( \text{counts}_j[d[A_i]] := \text{counts}_j[d[A_i]] + 0 \); // instantiates \( \text{counts}_j[d[A_i]] \)
      endif
    endfor
  endfor
  foreach \( x \) : index of instance of \( \text{counts}_1 \) do
    // computes entropy of binary split at \( x \)
    \( \text{Gain}[x] := p0 - \text{entropy}(D, A_i, x, \text{counts}_1, \ldots, \text{counts}_k) \);
  endfor
  best := arg(\text{findMax}(\text{Gain})));
  return best;
end

Figure 1: A modified version of selectSplittingAttribute() function for the C4.5 Algorithm. This version finds the best binary split for any continuous attribute.
The information gain obtained by using \( A_i \) with the binary split at \( \alpha \) is:

\[
Gain_{A_i, \alpha}(D) = \text{entropy}(D) - \text{entropy}_{A_i, \alpha}(D).
\]

**Finding best binary split.** The new version of the `selectSplittingAttribute()` function is in Figure 1.

- When attribute \( A_i \) is **continuous**, new `selectSplittingAttribute()` calls `findBestSplit()` function, also shown in Figure 1.
- To find the best binary split, we
  - scan the dataset \( D \) and determine the list of all values of \( A_i \).
    
    *Note, that while \( \text{dom}(A_i) \) is continuous, \( D \) contains finitly many distinct values of \( A_i \)!*
  - For each value \( x \) in of \( A_i \) from \( D \) find \( \text{entropy}_{A_i, x}(D) \).
  - Find \( x \) with the largest information gain and return it.

**Other adjustments to C4.5.** One more adjustment to C4.5 needs to be made.

- if a **categorical attribute** is selected to split \( D \) on the current step of the algorithm, this attribute is **removed from the attribute list** passed in the recursive calls to C4.5. (same as before)
- if a **continuous attribute** is selected to split \( D \) on the current step of the algorithm, this attribute is **kept in the attribute list** passed in the recursive calls to C4.5. (new)

**C4.5. and Overfitting**

**Overfitting.** Let \( D_{\text{training}} \) be a training set for a classification problem, and \( D_{\text{test}} \) be a test set. Let \( f \) be a classifier trained on \( D_{\text{training}} \).

\( f \) **overfits the data**, if there exists another classifier \( f' \) which has **lower accuracy** than \( f \) on \( D_{\text{training}} \) but **higher accuracy** than \( f \) on \( D_{\text{test}} \).

**Causes of overfitting:**

- **Noise in data.** (e.g., wrong class labels)
- **Randomness phenomena.** (training set is not representative of the application domain)
- **Complexity of model.** (too many attributes, some may not be needed for classification)
Dealing with overfitting. Two main approaches:

- **Pre-pruning** or **stopping early**. E.g., the *third termination condition in Algorithm C4.5* terminates tree construction early using the user-specified threshold parameter.

- **Post-pruning** or **pruning a constructed tree**. In this approach, the classification algorithm is allowed to *possibly overfit* the data, but a separate pruning algorithm will then check the classifier for overfitting.

### $k$-Nearest Neighbors Classification ($k$NN)

*C4.5* and **many other classification techniques** (Neural Nets, SVMs, Rule Induction) are *eager*: these techniques analyze the training set and construct a classifier *before any test data is read*.

The principle of **lazy evaluation** is to *postpone any data analysis until an actual question has been asked*.

In case of supervised learning, **lazy evaluation** means **not building a classifier** in advance of reading data from the test data set.

**$k$-Nearest Neighbors Classification algorithm** ($k$NN). $k$NN is a simple, but surprisingly robust **lazy evaluation** algorithm. The idea behind $k$NN is as follows:

- The input of the algorithm is a training set $D_{training}$, an instance $d$ that needs to be classified and an integer $k > 1$.
- The algorithm computes the *distance* between $d$ and every item $d' \in D$.
- The algorithm selects $k$ *most similar* or *closest* to $d$ records from $D$: $d_1, \ldots, d_k$, $d_i \in D$.
- The algorithm assigns to $d$ the class of the plurality of items from the list $d_1, \ldots, d_k$.

**Distance/similarity measures.** The distance (or similarity) between two records can be measured in a number of different ways.

**Note:** Similarity measures increase as the similarity between two objects increases. Distance measures decrease as the similarity between two objects increases.

1. **Euclidean distance.** If $D$ has continuous attributes, each $d \in D$ is essentially a point in $N$-dimensional space (or an $N$-dimensional vector). Euclidean distance:

\[
 d(d_1, d_2) = \sqrt{\sum_{i=1}^{n} (d_1[A_i] - d_2[A_i])^2},
\]

works well in this case.
2. **Manhattan distance.** If $D$ has ordinal, but not necessarily continuous attributes, Manhattan distance may work a bit better:

$$d(d_1, d_2) = \sum_{i}^{n} |d_1[A_i] - d_2[A_i]|.$$

3. **Cosine similarity.** Cosine distance between two vectors is the cosine of the angle between them. Cosine similarity ignores the amplitude of the vectors, and measures only the difference in their direction:

$$sim(d_1, d_2) = \cos(d_1, d_2) = \frac{d_1 \cdot d_2}{||d_1|| \cdot ||d_2||} = \frac{\sum_{i=1}^{n} d_1[A_i] \cdot d_2[A_i]}{\sqrt{\sum_{i=1}^{n} d_1[A_i]^2} \cdot \sqrt{\sum_{i=1}^{n} d_2[A_i]^2}}.$$

If $d_1$ and $d_2$ are colinear (have the same direction), $sim(d_1, d_2) = 1$. If $d_1$ and $d_2$ are orthogonal, $sim(d_1, d_2) = 0$.

**Ensemble Learning**

**Bagging**

**Bagging = Bootstrap aggregating.**

**Bootstrapping** is a statistical technique that one to gather many alternative versions of the single statistic that would ordinarily be calculated from one sample.

**Typical bootstrapping scenario.** (case resampling) Given a sample $D$ of size $n$, a bootstrap sample of $D$ is a sample of $n$ data items drawn randomly with replacement from $D$.

**Note:** On average, about 63.2% of items from $D$ will be found in a bootstrapping sample, but some items will be found multiple times.

**Bootstrap Aggregating for Supervised Learning.** Let $D$ be a training set, $|D| = N$. We construct a bagging classifier for $D$ as follows:

**Training Stage:** Given $D, k$ and a learning algorithm **BaseLearner**:

1. Create $k$ bootstrapping replications $D_1, \ldots, D_k$ of $D$ by using case resampling bootstrapping technique.
2. For each bootstrapping replication $D_i$, create a classifier $f_i$ using the **BaseLearner** classification method.

**Testing Stage:** Given $f_1, \ldots, f_k$ and a test record $d$:

1. Compute $f_1(d), \ldots f_k(d)$.
2. Assign as $class(d)$, the majority (plurality) class among $f_1(d), \ldots, f_k(d)$.

**Boosting**

**Boosting.** Boosting is a collection of techniques that generate an ensemble of classifiers in a way that each new classifier tries to correct classification errors from the previous stage.
Algorithm AdaBoost\((D, \text{BaseLearner}, k)\) begin

\begin{algorithmic}
  \State \textbf{foreach} \(d_i \in D\) do \(D_i(i) = \frac{1}{|D|}\);
  \For {\(t = 1\) to \(k\)} \hspace{1em} /main loop
  \State \(f_t := \text{BaseLearner}(D_t)\);
  \State \(e_t := \sum_{\text{class}(d_i) \neq f_t(d_i)} D_t(i)\);
  \hspace{2em} // \(f_t\) is constructed to minimize \(e_t\)
  \If {\(e_t > 0.5\)} \hspace{1em} // large error: redo
  \State \(k := k - 1\);
  \State \textbf{break};
  \EndIf
  \State \(a_t := \frac{1}{2} \ln \frac{1 - e_t}{e_t}\); \hspace{1em} //reweighting parameter
  \For {\(d_i \in D\)} \(D_{t+1}(i) := D_t(i) \cdot e^{-a_t \cdot \text{class}(d_i) \cdot f_t(d_i)}\); \hspace{1em} //reweigh each tuple in \(D\)
  \EndFor
  \State \(\text{Norm}_t := \sum_{i=1}^{|D|} D_{t+1}(i)\);
  \For {\(d_i \in D\)} \(D_{t+1}(i) := \frac{D_{t+1}(i)}{\text{Norm}_t}\); \hspace{1em} //normalize new weights
  \EndFor
  \EndFor
  \State \(f_{\text{final}}(.) := \text{sign}(\sum_{t=1}^k a_t \cdot f_t(.)\))
\end{algorithmic}

end

Figure 2: AdaBoost: an adaptive boosting algorithm. This version is for binary category variable \(Y = \{-1, +1\}\).

\textbf{Idea.} \textbf{Boosting} is applied to a specific classification algorithm called \textbf{BaseLearner}\(^1\).

Each item \(d \in D\) is assigned a weight. On first step, \(w(d) = \frac{1}{|D|}\). On each step, a classifier \(f_t\) is built. Any errors of classification, i.e, items \(d \in D\), such that \(f(d) \neq \text{class}(d)\) are given higher weight.

On the next step, the classification algorithm is made to "pay more attention" to items in \(D\) with higher weight.

The final classifier is constructed by weighting the votes of \(f_1, \ldots, f_k\) by their weighted classification error rate.

**AdaBoost.** The Adaptive \textbf{Boosting} algorithm [2] (AdaBoost) is shown in Figure 2.

\textbf{Weak Classifiers.} Some classifiers are designed to incorporate the weights of training set elements into consideration. But most, like \textbf{C4.5}, do not do so. In order to turn a classifier like \textbf{C4.5} into a \textbf{weak classifier} suitable for \textbf{AdaBoost}, this classifier can be updated as follows:

- On step \(t\), given the weighted training set \(D_t\), we \textbf{sample} \(D_t\) to build a training set \(D'_t\). The sampling process uses \(D_t(i)\) as the probability of selection of \(d_i\) into \(D'_t\) on each step.

\textbf{Voting}

When multiple classification algorithms \(A_1, \ldots, A_k\) are available, \textbf{direct voting} can be used to combine these classifiers.

\(^1\)It is also commonly called \textbf{weak classifier}.
Let $D$ be a training set, and $f_1, \ldots, f_k$ are the classifiers produced by $A_1, \ldots, A_k$ respectively on $D$. Then the combined classifier $f$ is constructed to return the class label returned by the plurality of classifiers $f_1, \ldots, f_k$.

**Random Forests**

Random Forests\[1\] are an extension of bagging. A bagging technique resamples the training set with replacement, but keeps all attributes in the dataset ”active” for each resampled training set.

Random Forests build a collection of decision trees, where each decision tree is built based on a subset of a training set and a subset of attributes.

In a nutshell, a Random Forests classifier works as follows:

1. **Input:** Let $D = \{d_1, \ldots, d_n\}$ be the training set, with $\text{class}(d_i)$ defined. Let $C = \{c_1, \ldots, c_k\}$ be the class attribute, and let $A = \{A_1, \ldots, A_N\}$ be the set of attributes for vectors from $D$, i.e., given $d \in D$, $d = (x_1, \ldots, x_M)$.

2. **Attribute selection parameter:** A number $m << M$ is fixed throughout the run of a random forest classifier. This number indicates how many attributes is selected to build each decision tree in a forest.

3. **Forest construction:** The classifier builds $N$ decision trees $T_1, \ldots, T_N$. Each decision tree is built by selecting a subsample of the training set, and a subset of the attributes.

4. **Single decision tree construction:** Decision tree $T_j$ is built as follows.
   
   (a) Build a set $D_j \subseteq D$ drawing random $k$ data points from $D$ with replacement.
   
   (b) Select $m$ random attributes $A_{j1}^1, \ldots, A_{jm}^j$ from $A$ without replacement.
   
   (c) Using a decision tree induction procedure (see below), build a decision tree $T_j$ for the training set $D_j$ restricted to attributes $A_{j1}^1, \ldots, A_{jm}^j$.
   
   Do not prune the decision trees.

5. **Classification process:** For each data point $d \in D$, (attempt to) classify $d$ by traversing trees $T_1, \ldots, T_N$ to discover classification decisions $c_1^1, \ldots, c_N^N$. Choose, as $\text{class}(d)$, the most frequently occurring in $c_1^1, \ldots, c_N^N$ class.

**Caveats.** A decision tree $T_j$ may not contain all possible values (paths) for some attribute. This means that some trees won’t be able to classify some of the data points in $D$. The simplest way to deal with this is to ignore.

**Decision tree induction procedures.** Both versions of ID3 (C4.5 without the pruning) and CART, a decision-tree induction algorithm that uses the Gini impurity instead of Information Gain-based measures, can be used.
The Gini impurity measure quantifies how often a randomly chosen and randomly labelled data point from a training set will be mislabelled.

Let \( D = \{d_1, \ldots, d_n\} \) be a training set.

Let \( C = \{c_1, \ldots, c_k\} \) be a class variable.

Let \( D_i = \{d \in D|\text{class}(d) = c_i\} \) be the set of all training set points from category \( c_i \).

Let \( f_i = |D_i| \).

The Gini impurity measure \( I_G \) is defined as follows:

\[
I_G(D) = \sum_{i=1}^{k} f_i \cdot (1 - f_i) = \sum_{i=1}^{k} k f_i - \sum_{i=1}^{k} k f_i^2 = 1 - \sum_{i=1}^{k} k f_i^2 = \sum_{i \neq j} f_i \cdot f_j.
\]

References
