Serializability Revisited

Goals of DBMS:

1. Ensure consistency of database states.
2. Process transactions efficiently.

Serial schedules ensure database consistency, but serial execution of transactions is inefficient.

Serializable schedules are defined in order to bridge the gap between consistency and efficiency.

- As their outcome must be equivalent to the result of some serial schedule, serializable schedules are guaranteed to preserve consistency of the database.
- As they are not required to be serial, serializable schedules allow for interleaving of actions from different transactions.

Question 1: How much concurrency is allowed by serializable schedules?

Given transaction $T$, we denote as $\text{dom}(T)$ the set of all database objects that $T$ accesses (both in read and write modes).

Proposition 1 Let $T_1$ and $T_2$ be two transactions such that $\text{dom}(T_1) \cap \text{dom}(T_2) = \emptyset$. Then

1. Both serial schedules $T_1;T_2$ and $T_2;T_1$ result in the same database state.
2. **Any valid schedule over** $T_1$ **and** $T_2$ **is serializable.**

**Proof.** (sketch)

1. To prove that both serial schedules result in the same database state, we examine the resulting value of each database object in $D = dom(T_1) \cup dom(T_2)$. If $A \in D$, then by our assumption, either $A \in dom(T_1)$ and $A \notin dom(T_2)$ or vice versa. Therefore, its value gets modified by only one transaction, and does not change throughout the execution of the second transaction.

2. We notice that any schedule over the set $\{T_1, T_2\}$ has no RW, WR or WW conflicts. Then the statement follows from the following, more general lemma.

**Lemma 1** Let $S$ be a schedule without conflicts over two committed transactions $T_1$ and $T_2$. Then $S$ is a serializable schedule.

**Proof.** (sketch) We need to show that the final state of the database after $S$ is executed is equivalent to the final state of the database after either $T_1; T_2$ or $T_2; T_1$.

We consider all objects in $dom(T_1) \cup dom(T_2)$. Two possibilities arise:

1. $dom(T_1) \cap dom(T_2) \neq \emptyset$. In this case, one or more objects $T_1$ and $T_2$ access are the same. Given an object $A \in dom(T_1) \cap dom(T_2)$, there are only two possibilities: (i) either both $T_1$ and $T_2$ have read-only access to $A$ or (ii) one of the transactions accesses $A$ after the other transaction has committed.

   If there is at least one object $A \in dom(T_1) \cap dom(T_2)$ for which (ii) holds, then we claim that this object uniquely defines the serial schedule to which $S$ is equivalent. Indeed, assume that $A$ is accessed first by $T_1$ and then – by $T_2$ after $T_1$ commits.

   We claim that $S$ in this case is equivalent to $T_1; T_2$. Clearly, the value of $A$ after $S$ is equivalent to the value of $A$ after $T_1; T_2$. The values of all objects $B \in dom(T_1) \cup dom(T_2) - dom(T_1) \cap dom(T_2)$ after $S$ will match those after $T_1; T_2$ (see Proposition 1.1).

   Let $A' \in dom(T_1) \cap dom(T_1)$. If both $T_1$ and $T_2$ only read $A'$ then its value after $S$ will be the same as its value after $T_1; T_2$. We need to show that it is impossible for transaction $T_2$ to modify $A'$s value before $T_1$. This is indeed so. We know that $T_1$ accesses $A'$s value.

   If $T_1$ writes $A'$s value, $T_2$ cannot access $A'$ until after $T_1$ commits (as $S$ is conflict-free), therefore the final value of $A'$ will be the same as in the serial schedule $T_1; T_2$.

   If $T_1$ reads $A'$s value, $T_2$ cannot write it until $T_1$ commits (otherwise, a conflict would be registered in $S$). This, however means that again, the final value of $A'$ is determined by $T_2$, and therefore is equal to that of $T_1; T_2$ serial schedule, which proves the first part of the lemma.
2. \( \text{dom}(T1) \cap \text{dom}(T2) = \emptyset \). This is really Proposition 1.2. From Proposition 1.1 we know that both \( T1;T2 \) and \( T2;T2 \) yield the same database state. Simple analysis of the values of each object in \( \text{dom}(T1) \cup \text{dom}(T2) \) after \( S \) ends shows that these values will be the same as those in any serial schedule.

From the two result above, we notice that

- **Serializable schedules** allow for arbitrary interleaving of the transactions that access **completely different sets of database objects**.
- **Serializable schedules** do allow certain degree of interleaving when transactions access the same object.
- All **conflict-free schedules** are serializable.

One would want to know if the reverse of the latter is true:

**Question 2**: Are all serializable schedules conflict-free?

The answer to this question is **NO**.

Below is an example of a **serializable schedule** that is **not** conflict-free (contains an unrepeatable read).

<table>
<thead>
<tr>
<th></th>
<th>T1</th>
<th>T2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R(A) )</td>
<td>( W(A) )</td>
</tr>
<tr>
<td>commit</td>
<td>commit</td>
<td>commit</td>
</tr>
</tbody>
</table>

The schedule above is equivalent to serial schedule \( T1;T2 \). There is an **RW** conflict in the schedule, but it never “materializes” to affect the outcome.

**Locking**

**Goal of DBMS (revised):**

Ensure **serializability** of all schedules.

To achieve this goal, DBMS may want to ensure the following properties of its schedules:

- If some transaction \( T \) has read some object \( A \), **no** transaction \( T' \) can write \( A \) until \( T \) commits or aborts.
- If some transaction \( T \) has written some object \( A \), **no** transaction \( T' \) can access \( A \) until \( T \) aborts or commits.
Object Locking has been proposed as the way to assure these properties of the schedules.

**Lock:** permission by a DBMS to a transaction to *access* the content of a particular database object.

**Shared Lock:** permission to *read* the value of the object. More than one transaction can hold a **shared lock** on the same object at the same time.

**Exclusive Lock:** permission to *write* the value of the object. At most one transaction can hold an **exclusive lock** on an object at a time, and no **shared locks** are allowed by other transactions on an object for which an **exclusive lock** exists.

**New Rules For Transaction Processing**

- Before accessing a database object, any transaction must request an appropriate lock on it.
- If the lock is granted by the DBMS, the transaction may proceed.
- If the lock cannot be granted immediately, the lock request is queued and the transaction is suspended until the lock can be granted.
- Transaction must release all the locks it holds before it terminates (commits or aborts).

**Notation:** $S(A)$ – request for a **shared lock** on object $A$. $X(A)$ – request for an **exclusive lock** on object $A$. $U(A)$ – request to release current lock on the object $A$.

**Note:** We assume that abort and commit commands result in automatic release of all locks held by a transaction at that time, therefore, we do not specify all unlock requests explicitly, unless necessary.

Scheduling with locking is illustrated in the following example:

<table>
<thead>
<tr>
<th></th>
<th>$T1$</th>
<th>$T2$</th>
<th>$T3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S(A)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R(A)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>commit</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X(A)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R(A)$</td>
<td>$X(B)$</td>
<td>$S(A)$</td>
</tr>
<tr>
<td></td>
<td>$U(A)$</td>
<td>$R(A)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$W(A)$</td>
<td>$U(A)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R(A)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$W(A)$</td>
<td>$U(A)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>commit</td>
<td>$W(A)$</td>
<td></td>
</tr>
</tbody>
</table>

4
Figure 1: Non-serializable schedule with locking.

- Locking by itself does not guarantee serializability of schedules.

This is illustrated by the schedule in Figure 1

In this schedule, the value of object $A$ in the final state is written by $T_2$ and the value of object $B$ – by $T_1$, therefore it is not a serializable schedule.

**Locking Mechanisms: 2-Phase Locking**

**Locking Mechanism**: set of rules for locking and unlocking objects.

We want the locking mechanism to produce only serializable schedules.

**2-Phase Locking (2PL)**

2-Phase Locking (2PL) works according to the following two rules:

1. If a transaction $T$ wants to read/write an object $A$, it must first request a shared/exclusive lock on $A$.

2. Once a transaction released one lock it cannot request any additional locks.

Informally, with 2-Phase Locking, the life of any transaction consists of two periods (phases): (i) the period during which the transaction acquires new locks (growing) and (ii) the period during which the transaction releases its locks (shrinking).

It is easy to see that schedule from Figure 1 does not satisfy the conditions of 2PL as transaction $T_1$ acquires a lock on $B$ after it has released its lock on $A$. 
<table>
<thead>
<tr>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_1$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(A)$</td>
<td>$X(A)$</td>
<td>$X(A)$</td>
<td>$X(A)$</td>
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<tr>
<td>$X(B)$</td>
<td>$X(B)$</td>
<td>$X(B)$</td>
<td>$X(B)$</td>
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<tr>
<td>$W(A)$</td>
<td>$W(A)$</td>
<td>$W(A)$</td>
<td>$W(A)$</td>
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<tr>
<td>$U(A)$</td>
<td>$W(B)$</td>
<td>$W(B)$</td>
<td>$W(B)$</td>
</tr>
<tr>
<td>$W(B)$ commit</td>
<td></td>
<td>$W(B)$ commit</td>
<td></td>
</tr>
<tr>
<td>commit</td>
<td></td>
<td>commit</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Schedules that follow 2PL (left) and Strict 2PL (right) locking mechanisms.

**Strict 2-Phase Locking (Strict 2PL)**

**Strict 2-Phase Locking** is a modification of 2-Phase Locking which disallows nontrivial shrinking phase in any transaction. It can be described as follows:

1. If a transaction $T$ wants to read/write an object $A$, it must first request a shared/exclusive lock on $A$.

2. Once a transaction have acquired a lock it cannot release it until it commits or aborts.

Figure 2 contains the examples that illustrate 2PL and Strict 2PL locking mechanisms. The schedule on the left satisfies 2-Phase Locking requirements, but will not be accepted according to Strict 2-Phase Locking. The schedule on the left satisfies both 2PL and Strict 2PL mechanisms.

**Properties of 2 Phase and Strict 2 Phase Locking**

**Questions**

- Want mechanisms for producing serializable schedules.
- Locking and locking mechanisms required.
- 2 Phase Locking and Strict 2 Phase Locking.

**Question 1** Are the schedules produced by 2PL/Strict 2PL serializable?

**Question 2** Are all serializable schedules produced by 2PL (Strict 2PL)?
**Question 3** What is the difference between 2PL and Strict 2PL?

**Question 4** How do we characterize the schedules produced by 2PL / Strict 2PL?

**Conflict Serializability**

Let us consider for now only schedules consisting of committed transactions.

**Dependency (Serializability) Graph.**

Let $S$ be a schedule over the set of transactions $T = \{T_1, \ldots, T_N\}$. A dependency graph of $S$, denoted $G_S$, is defined as follows:

- The set of nodes of $G_S$ is $T$.
- $G_S$ has an edge from $T_i$ to $T_j$ labeled with a database object $A$ if
  1. both $T_i$ and $T_j$ access some object $A$;
  2. at least one of these accesses is a write;
  3. no other transaction accesses $A$ between $T_i$’s and $T_j$’s accesses.

**Conflict Equivalence.**

Two schedules $S_1$ and $S_2$ over the set of transactions $T$ are conflict-equivalent iff $G_{S_1} = G_{S_2}$.

**Conflict Serializability.**

A schedule $S$ is conflict-serializable iff it is conflict-equivalent to some serial schedule.

Figure 3 shows three different schedules for the same set of transactions $T = \{T_1, T_2\}$. Schedule $S_1$ is not serializable as at the end $A$ will have value set by $T_2$ and $B$ will have value set by $T_1$. Schedule $S_3$ is serial. To see that schedule $S_2$ is conflict-serializable, we construct the dependency graphs $G_{S_1}$, $G_{S_2}$ and $G_{S_3}$ (see Figure 4).

**Relationships Between Schedule Types**

**Theorem 1** A conflict serializable schedule over a static database is serializable.

The requirement that the database is static, i.e., no new objects created and no existing objects deleted while the schedule is executed is important. We will discuss this requirement and predicate locks later in the course.

The inverse of Theorem 1 is not true as manifested in the counterexample on Figure 5. Here, the schedule depicted is equivalent to the serial schedule $T_1; T_2; T_3$ (because $T_3$ blindly overwrites the actions of $T_1$ and $T_2$) but it
<table>
<thead>
<tr>
<th>T1</th>
<th>T2</th>
<th>T1</th>
<th>T2</th>
<th>T1</th>
<th>T2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(A)</td>
<td>S(C)</td>
<td>X(A)</td>
<td>X(B)</td>
<td>S(C)</td>
<td>commit</td>
</tr>
<tr>
<td>X(B)</td>
<td>R(C)</td>
<td>R(C)</td>
<td>commit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W(A)</td>
<td>W(A)</td>
<td>U(A)</td>
<td>X(B)</td>
<td>commit</td>
<td></td>
</tr>
<tr>
<td>U(A)</td>
<td>X(A)</td>
<td>W(B)</td>
<td>W(B)</td>
<td>commit</td>
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<td>commit</td>
<td>commit</td>
<td>commit</td>
<td>commit</td>
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</tr>
</tbody>
</table>

Figure 3: Non-serializable, conflict-serializable and serial schedules.

Figure 4: Dependency Graphs for schedules S1, S2 and S3.
Figure 5: A serializable schedule that is not conflict-serializable.

is not conflict-equivalent to any serial schedule (as can be verified by building appropriate graphs).

**Note:** Conflict-serializability is a syntactic property of a schedule while serializability is a semantic property. Conflict-serializability is a stronger property.

**Lemma 2**

1. The dependency graph for a serial schedule is acyclic.
2. A schedule is conflict-serializable iff its dependency graph is acyclic.

**Theorem 2** Let $S$ be a schedule generated according to a 2 Phase Locking Mechanism over a set of transactions $T = \{T_1, \ldots, T_N\}$. Then $G_S$ is acyclic.

From Lemma 2 and Theorem 2 we infer

**Theorem 3** Any schedule produced by 2 Phase Locking mechanism is conflict-serializable (and hence, serializable).

**2 Phase Locking vs. Strict 2 Phase Locking**

**Theorem 4** Every schedule conforming to Strict 2 Phase Locking also conforms to 2 Phase Locking.

**Question 5:** What schedules are excluded by Strict 2PL?

**Strict Schedules.**

A schedule $S$ over the set of transactions $T = T_1, \ldots, T_N$ is called strict iff any value written by any transaction $T_i$ to any database object $A$ does not get accessed by other transactions until $T_i$ terminates (commits or aborts).
T1 | T2
---|---
X(A) | X(A)
R(A) | R(A)
W(A) | W(A)
X(B) | X(C)
U(A) | 
R(B) | W(C)
| commit
| abort

Figure 6: 2 Phase Locking is insufficient to ensure strictness of the schedules.

**Theorem 5** Any schedule produced by Strict 2 Phase Locking Mechanism is strict.

**Question 6:** Why is strictness of schedules important.

In a non-strict schedule, if a transaction aborts, it may cause other transactions to abort.

Figure 6 illustrates (i) the problems with non-strict schedules in the presence of aborted transactions and (ii) that 2 Phase Locking may produce non-strict schedules.

When transaction T2 commits, the changes it makes to data become permanent. However, when T1 aborts, the changes it made to object A should be rolled back. However, the changes made by T2 to A and (possibly) C depend on the value of A as written by T1. Therefore, after T1 aborts, the results of T2 become stale and have to be undone.

**Bottomline:** Strict 2 Phase Locking ensures that no transaction has access to data that may become stale, by enforcing strictness of the schedules generated by it. Because of this, despite the extra limitations Strict 2PL puts on interleaving of transactions, it is preferable to simple 2 Phase Locking.