Poisson Process

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Data 401

The Poisson process is commonly used to model random “arrivals”, like cars crossing an intersection or customer purchases at a store.

| x x x x | x x x x x | → t |
| 0 1 2 3 4 5 |

| x x x x x | x x x x x | → t |
| 0 1 2 3 4 5 |

| x x x x x | x x x x x | x → t |
| 0 1 2 3 4 5 |

The Poisson process is characterized by two properties:

1. The number of arrivals in the interval \([t_0, t_1]\) is a Poisson(\(\lambda(t_1 - t_0)\)) random variable.

2. The numbers of arrivals in disjoint intervals are independent.

\(\lambda\) is a rate parameter representing the expected number of arrivals per unit of time.
Example
Suppose that requests to a server follow a Poisson process at a rate of 2 calls per minute.

(a) What is the probability that there are exactly 8 requests over a 5-minute period?

(b) What is the probability that there are exactly 8 requests over a 5-minute period, given that there were 2 requests in the first minute?

Interarrival Times in a Poisson Process

The **interarrival times** are the times $T_1, T_2, \ldots$ between successive arrivals.

What is the distribution of the interarrival times?
Let’s work out the distribution of $T_1$. 
Distribution of Interarrival Times

A Peek Ahead

This is a data science class, so we are interested in using the Poisson process to model data.

Suppose the first 5 purchases at an online store happen at:

8:01:20, 8:04:26, 8:04:56, 8:06:50, 8:11:20.

and you would like to model the purchases using a Poisson process.

How would you estimate the rate parameter $\lambda$ of the Poisson process?