Evaluating Methods: Bias

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Data 401
**Statistical Framework for Evaluating Methods**

Given data, we assume a probability model for the data and estimate the parameters in that model.

\[
\theta_0 \xrightarrow{p_{\theta_0}(x)} \text{Data } x \xrightarrow{L(\theta) = p_\theta(x)} \hat{\theta}
\]

How do we know if the estimate is any good or not?

Statisticians assume that there is some true underlying model that generated the data and study how \( \hat{\theta} \) compares to \( \theta_0 \).

Notice that there are 3 “\( \theta \)”s floating around:

- \( \theta_0 \) is the true value of the parameter
- \( \theta \) is the variable that we optimize over to get the MLE
- \( \hat{\theta} \) is the estimated value of the parameter
Bias

One metric that we can calculate is the bias:

\[
\text{bias} = E[\hat{\theta}] - \theta_0.
\]

Example

For example, let’s model data \(X_1, \ldots, X_n\) as i.i.d. Poisson(\(\mu\)).

We saw that the MLE is \(\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} X_i\). Is this a good estimator?

Assume that \(X_1, \ldots, X_n\) are actually generated from Poisson(\(\mu_0\)).

\[
E[\hat{\mu}] = E\left[\frac{1}{n} \sum_{i=1}^{n} X_i\right] = \frac{1}{n} \sum_{i=1}^{n} E[X_i] = \frac{1}{n} \sum_{i=1}^{n} \mu_0 = \frac{1}{n} n \mu_0 = \mu_0
\]

So the bias of the MLE in this case is \(E[\hat{\mu}] - \mu_0 = 0\).

We say this estimator is unbiased.
The Bias of Linear Regression

Given data \((x_1, Y_1), \ldots, (x_n, Y_n)\), we assumed the simple linear regression model (without intercept) \(Y_i \sim \text{Normal}(x_i \beta, \sigma^2)\) and calculated the MLE of \(\beta\) as

\[
\hat{\beta} = \frac{\sum_{i=1}^{n} x_i Y_i}{\sum_{i=1}^{n} x_i^2}.
\]

What is the bias of \(\hat{\beta}\)?

The argument generalizes to multiple regression.