CPE/CSC 481: Knowledge-Based Systems

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Overview Approximate Reasoning

- Motivation
- Objectives
- Approximate Reasoning
  - Variation of Reasoning with Uncertainty
  - Commonsense Reasoning
- Fuzzy Logic
  - Fuzzy Sets and Natural Language
  - Membership Functions
  - Linguistic Variables
- Important Concepts and Terms
- Chapter Summary

Motivation

- reasoning for real-world problems involves missing knowledge, inexact knowledge, inconsistent facts or rules, and other sources of uncertainty
- while traditional logic in principle is capable of capturing and expressing these aspects, it is not very intuitive or practical
  - explicit introduction of predicates or functions
- many expert systems have mechanisms to deal with uncertainty
  - sometimes introduced as ad-hoc measures, lacking a sound foundation

Objectives

- be familiar with various approaches to approximate reasoning
- understand the main concepts of fuzzy logic
  - fuzzy sets
  - linguistic variables
  - fuzzification, defuzzification
  - fuzzy inference
- evaluate the suitability of fuzzy logic for specific tasks
  - application of methods to scenarios or tasks
- apply some principles to simple problems

Approximate Reasoning

- inference of a possibly imprecise conclusion from possibly imprecise premises
- useful in many real-world situations
  - one of the strategies used for “common sense” reasoning
  - frequently utilizes heuristics
  - especially successful in some control applications
- often used synonymously with fuzzy reasoning
- although formal foundations have been developed, some problems remain

Approaches to Approximate Reasoning

- fuzzy logic
  - reasoning based on possibly imprecise sentences
- default reasoning
  - in the absence of doubt, general rules ("defaults) are applied
  - default logic, nonmonotonic logic, circumscription
- analogical reasoning
  - conclusions are derived according to analogies to similar situations

Advantages of Approximate Reasoning

- common sense reasoning
  - allows the emulation of some reasoning strategies used by humans
- concise
  - can cover many aspects of a problem without explicit representation of the details
- quick conclusions
  - can sometimes avoid lengthy inference chains

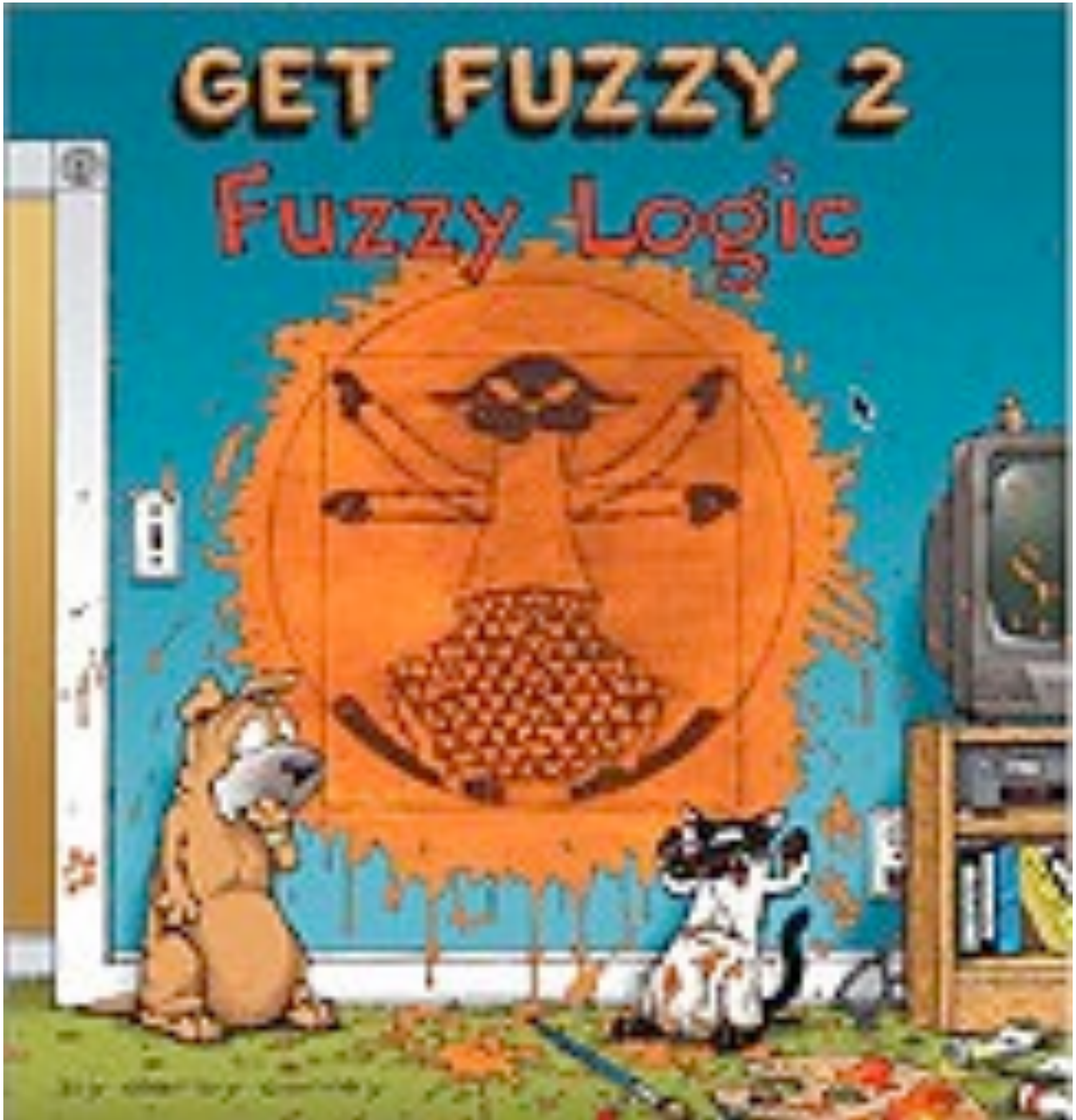
Problems of Approximate Reasoning

- non-monotonicity
  - inconsistencies in the knowledge base may arise as new sentences are added
  - sometimes remedied by truth maintenance systems
- semantic status of rules
  - default rules often are false technically
- efficiency
  - although some decisions are quick, such systems can be very slow
    - especially when truth maintenance is used

Fuzzy Logic

- approach to a formal treatment of uncertainty
- relies on quantifying and reasoning through natural language
  - linguistic variables
    - used to describe concepts with vague values
  - fuzzy qualifiers
    - a little, somewhat, fairly, very, really, extremely
  - fuzzy quantifiers
    - almost never, rarely, often, frequently, usually, almost always
    - hardly any, few, many, most, almost all

Fuzzy Logic in Entertainment

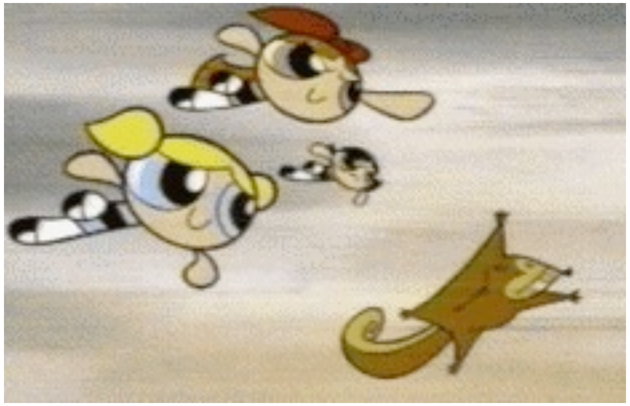


Thursday, February 9, 12


SUPER FURRY
ANIMALS
FUZZY LOGIC

- Powerpuff Girls episode
  - Fuzzy Logic: Beastly bumpkin Fuzzy Lumpkins goes wild in Townsville and only the Powerpuff Girls—with some help from a flying squirrel—can teach him to respect other people's property.

http://en.wikipedia.org/wiki/Fuzzy_Logic_(Powerpuff_Girls_episode)

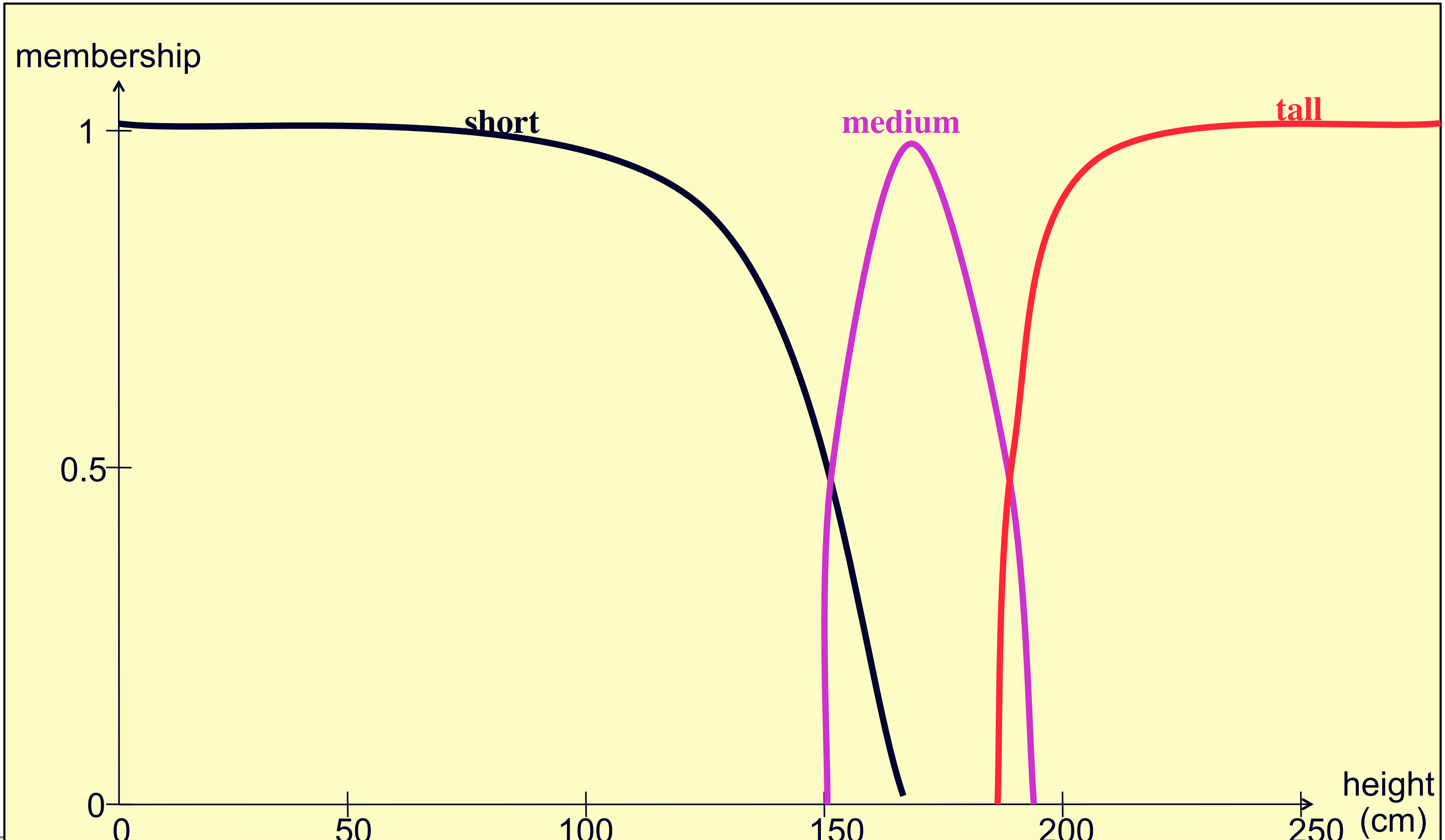


http://www.templelooters.com/powerpuff/PPG4.htm

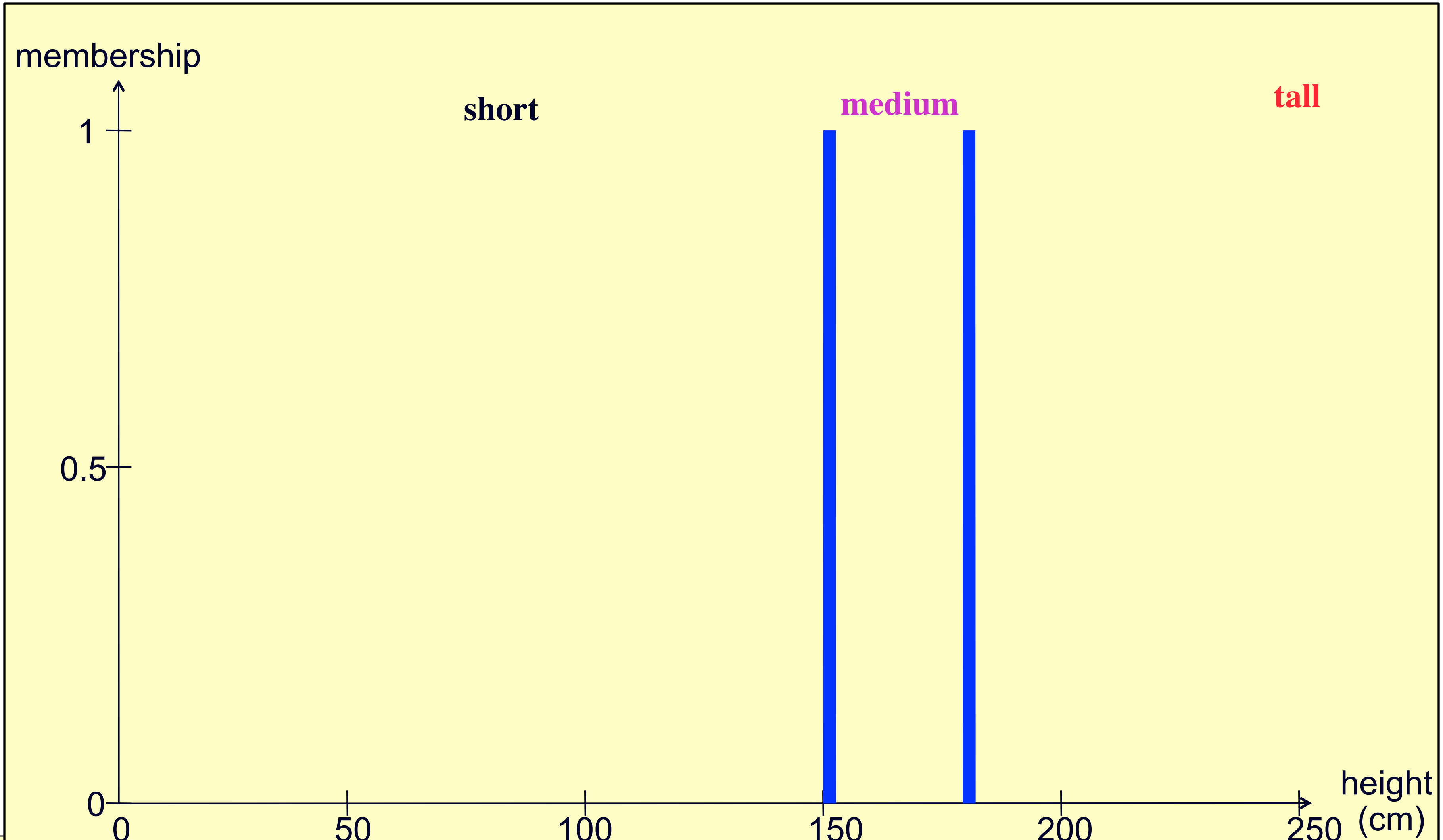
Fuzzy Sets

- categorization of elements x_i into a set S
  - described through a membership function
    $\mu(s): \quad x \rightarrow [0,1]$
    - associates each element xi with a degree of membership in S:
      - 0 = no membership
      - 1 = full membership
      - values in between indicate how strongly an element is affiliated with the set

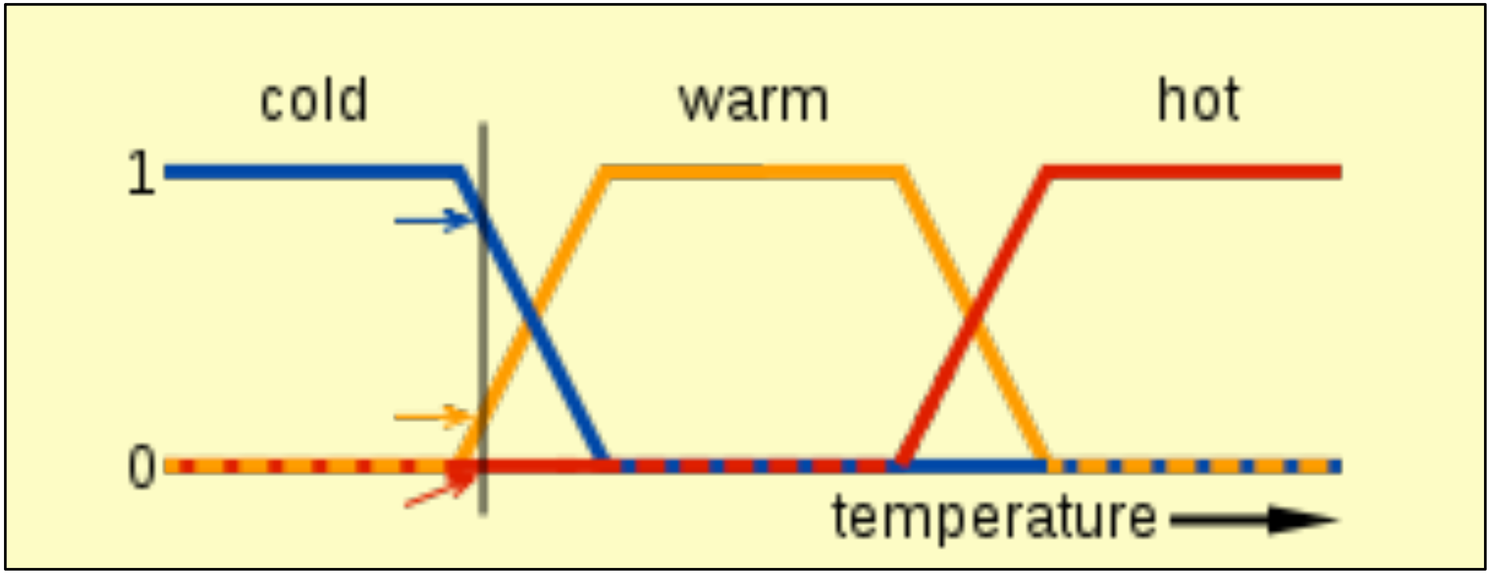
Fuzzy Set Example



Fuzzy vs. Crisp Set



Fuzzy Logic Temperature



http://commons.wikimedia.org/wiki/
File:Warm_fuzzy_logic_member_function.gif

Possibility Measure

- degree to which an individual element x is a potential member in the fuzzy set S
  $Poss\{x \in S\}$
- combination of multiple premises with possibilities
  - various rules are used
  - a popular one is based on *minimum* and *maximum*
    - $Poss(A \wedge B) = min(Poss(A),Poss(B))$
    - $Poss(A \vee B) = max(Poss(A),Poss(B))$

Possibility vs. Probability

- *possibility*
  - refers to *allowed* values
- *probability*
  - expresses *expected* occurrences of events

- Example: rolling a pair of dice
  - X is an integer in U = {2,3,4,5,6,7,8,9,19,11,12}
  - probabilities

    p(X = 7) = 2*3/36 = 1/6

    7 = 1+6 = 2+5 = 3+4
  - possibilities

    Poss{X = 7} = 1

    the same for all numbers in U

Fuzzification

- extension principle
- defines how a value, function or set can be represented by a corresponding fuzzy membership function
- extends the known membership function of a subset to
  - a specific value
  - a function
  - the full set

function	$f: X \rightarrow Y$
membership function	μA for a subset $A \subseteq X$
extension	$\mu f(A) (f(x)) = \mu A(x)$

[Kasabov 1996]

De-fuzzification

- converts a fuzzy output variable into a single-value variable
- widely used methods are
  - center of gravity (COG)
    - finds the geometrical center of the output variable
  - mean of maxima
    - calculates the mean of the maxima of the membership function

[Kasabov 1996]

Fuzzy Logic Translation Rules

- describe how complex sentences are generated from elementary ones
- modification rules
  - introduce a linguistic variable into a simple sentence
    - e.g. “John is very tall”
- composition rules
  - combination of simple sentences through logical operators
    - e.g. condition (if ... then), conjunction (and), disjunction (or)
- quantification rules
  - use of linguistic variables with quantifiers
    - e.g. most, many, almost all
- qualification rules
  - linguistic variables applied to truth, probability, possibility
    - e.g. very true, very likely, almost impossible

Fuzzy Probability

- describes probabilities that are known only imprecisely
  - e.g. fuzzy qualifiers like very likely, not very likely, unlikely
  - integrated with fuzzy logic based on the qualification translation rules
    - derived from Lukasiewicz logic
      - multi-valued logic

Fuzzy Inference Methods

- how to combine evidence across fuzzy rules
  - Poss(B|A) = min(1, (1 - Poss(A)+ Poss(B)))
    - implication according to Max-Min inference
  - also Max-Product inference and other rules
  - formal foundation through Lukasiewicz logic
    - extension of binary logic to infinite-valued logic

Fuzzy Inference Rules

- principles that allow the generation of new sentences from existing ones
  - the general logical inference rules (modus ponens, resolution, etc)
    are not directly applicable
- examples
  - entailment principle

$$\frac{\begin{array}{l}X \text{ is } F \\ F \subset G\end{array}}{X \text{ is } G}$$

  - compositional rule

$$\frac{\begin{array}{l}X \text{ is } F \\ (X,Y) \text{ is } R\end{array}}{Y \text{ is } \max(F,R)}$$

X, Y are elements

F, G, R are relations

Example Fuzzy Reasoning 1

- bank loan decision case problem
  - represented as a set of two rules with tables for fuzzy set definitions
    - fuzzy variables

      `CScore, CRatio, CCredit, Decision`
    - fuzzy values

      `high score, low score,`
      `good_cc, bad_cc, good_cr, bad_cr,`
      `approve, disapprove`

```
Rule 1: If (CScore is high) and (CRatio is good_cr)
            and (CCredit is good_cc)
        then (Decision is approve)
Rule 2: If (CScore is low) and (CRatio is bad_cr)
            or (CCredit is bad_cc)
        then (Decision is disapprove )
```

[Kasabov 1996]

Example Fuzzy Reasoning 2

- tables for fuzzy set definitions

CScore	150	155	160	165	170	175	180	185	190	195	200
high	0	0	0	0	0	0	0.2	0.7	1	1	1
low	1	1	0.8	0.5	0.2	0	0	0	0	0	0

CCredit	0	1	2	3	4	5	6	7	8	9	10
good_cc	1	1	1	0.7	0.3	0	0	0	0	0	0
bad_cc	0	0	0	0	0	0	0.3	0.7	1	1	1

CRatio	0.1	0.3	0.4	0.41	0.42	0.43	0.44	0.45	0.5	0.7	1
good_cc	1	1	0.7	0.3	0	0	0	0	0	0	0
bad_cc	0	0	0	0	0	0	0	0.3	0.7	1	1

Decision	0	1	2	3	4	5	6	7	8	9	10
approve	0	0	0	0	0	0	0.3	0.7	1	1	1
disapprove	1	1	1	0.7	0.3	0	0	0	0	0	0

[Kasabov 1996]

Advantages and Problems of Fuzzy Logic

- advantages
  - foundation for a general theory of commonsense reasoning
  - many practical applications
  - natural use of vague and imprecise concepts
  - hardware implementations for simpler tasks
- problems
  - formulation of the task can be very tedious
  - membership functions can be difficult to find
  - multiple ways for combining evidence
  - problems with long inference chains
  - efficiency for complex tasks

Important Concepts and Terms

- approximate reasoning
- common-sense reasoning
- crisp set
- default reasoning
- defuzzification
- extension principle
- fuzzification
- fuzzy inference
- fuzzy rule
- fuzzy set
- fuzzy value
- fuzzy variable
- imprecision
- inconsistency
- inexact knowledge
- inference
- inference mechanism
- knowledge
- linguistic variable
- membership function
- non-monotonic reasoning
- possibility
- probability
- reasoning
- rule
- uncertainty

Summary Approximate Reasoning

- attempts to formalize some aspects of common-sense reasoning
- fuzzy logic utilizes linguistic variables in combination with fuzzy rules and fuzzy inference in a formal approach to approximate reasoning
  - allows a more natural formulation of some types of problems
  - successfully applied to many real-world problems
  - some fundamental and practical limitations
    - semantics, usage, efficiency