Chapter Overview

Inexact Reasoning

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Chapter Review

Introduction

reasoning under uncertainty
and with inexact knowledge

heuristics
ways to mimic heuristic knowledge processing methods of experts

empirical associations
based on limited observations

probabilities
objective (frequency counting)
vs. subjective (human-oriented)

reproducability
can the observations be repeated, and will they deliver the same result?

approaches
numerical, or symbol-oriented

constraints
often time and money

exact reasoning, in contrast to inexact reasoning, is based on *deductive* approaches to logic

Objectives

for dealing with uncertainty and inexact knowledge

expressiveness
Can concepts used by humans be represented adequately? Can the confidence of experts in their choices be expressed?

comprehensibility
How difficult is it to understand the representation and the evaluation?

soundness
Are probability laws required (sum of conditional probabilities = 1), or is a relevance ranking sufficient?

consistency
similar results for similar inputs

example data
Are large quantities of historic data needed?

reasoning
long inference chains

computational complexity
Are the required calculations feasible?

portability
Can the method be used with any system and application?

Sources of Uncertainty

and inexact information

data
missing data
unreliable, ambiguous
inconsistent or imprecise representation of data
skewed by the user's perception ("best guess")
derived from defaults

expert knowledge
inconsistency between different experts
plausible: "best guess" of the expert
statistical associations observed by the expert
limited applicability

knowledge representation
represented knowledge doesn't exactly model the real system

inference process

- deductive: the application of a rule is formally correct, but the result is wrong
- inductive: new conclusions are obtained in an uncertain or inexact way
- other reasoning methods

Uncertainty and Rules

individual rules

- errors (see previous transparency)
- likelihood of evidence
  for each premise, and for the conclusion
- combination of evidence
  from multiple premises

conflict resolutions

- explicit priority provided by the expert
  relative or absolute ranking of rules
- implicit priority derived from rule properties
  specificity of patterns
  recency of facts matching patterns
  ordering of patterns (lexicographic, means end)
  order in which rules are entered

compatibility

- contradiction of rules
- subsumption
- redundancy
- missing rules
- data fusion

Basics of Probability Theory

mathematical approach for processing uncertain information

sample space set $X = \{x_1, x_2, \ldots, x_n\}$
collection of all possible events
can be discrete or continuous

probability number $P(x_i)$
likelihood of an event x_i to occur

- non-negative value from [0,1]
- total probability of the sample space is 1
- for mutually exclusive events, the probability for at least one of them is the sum of their individual probabilities
- *experimental* probability (a posteriori) based on the frequency of events
- *subjective* probability based on estimates of experts

compound probabilities

- for *independent* events that do not affect each other in any way
- *joint* probability (intersection) of two independent events A and B

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = P(A) * P(B)$$

where $n(S)$ is the number of elements in S

- *union* probability of two independent events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - P(A) \times P(B)$$

where $n(S)$ is the number of elements in S

conditional probabilities

- for *dependent* events that affect each other in some way
- *conditional* probability of event A given that event B has already occured

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Advantages and Problems

+ formal foundation

+ reflection of reality (a posteriori)

- may be inappropriate (future different from past)

- inexact or incorrect (subjective probabilities)

- some knowledge is represented implicitly

Bayesian Approaches

derive the probability of a cause given a symptom

inverse probability also *a posteriori* prob.
inverse to *conditional* probability probability of an earlier event given that a later one occurred

Bayes' rule for single hypothesis, single event

$$P(H|E) = \frac{P(E|H) * P(H)}{p(E)}$$

or

$$P(H|E) = \frac{P(E|H) * P(H)}{P(E|H) * P(H) + P(E|\neg H) * P(\neg H)}$$

where H is a hypothesis, and E and event

Bayes' rule for multiple hypotheses, multiple events
posterior probability of hypothesis h_i from evidence $E_1, \ldots, E_n$

$$P(H_i|E_1, E_2, \ldots E_n) = \frac{P(E_1, E_2, \ldots E_n|H) * P(H)}{p(E_1, E_2, \ldots E_n)}$$

or

$$P(H|E) =$$

$$\frac{P(E_1|H_i) * P(E_2|H_i) * \ldots * P(E_n|H_i) * P(H_i)}{\sum_{k=1}^{m} P(E_1|H_k) * P(E_2|H_k) * \ldots * P(E_n|H_k) * P(H_k}$$

where the pieces of evidence E_i are independent

Advantages and Problems

+ sound theoretical foundation

+ well-defined semantics for decision making

- require large amounts of probability data

- independence assumption of evidences frequently not valid

- sources of prior and conditional probabilities: statistics (sufficient sample sizes), human experts (consistent, comprehensive, trustworthy)

- relationship between hypothesis and evidence is reduced to a number

- explanations difficult to provide to the user

Certainty Factors

alternative to Bayesian methods

basic idea

denotes the belief in a hypothesis H given that some pieces of evidence E_i are observed
does not make any statement about the belief if no evidence is present (in contrast to Bayesian methods)

measure of belief

degree to which hypothesis H is supported by evidence E

$$MB(H,E) = \begin{cases} 1 & \text{if} P(H) = 1 \\ \frac{P(H|E) - P(H)}{1 - P(H)} & \text{otherwise} \end{cases}$$

measure of disbelief

degree to which doubt in hypothesis H is supported by evidence E

$$MD(H,E) = \begin{cases} 1 & \text{if} P(H) = 0 \\ \frac{P(H) - P(H|E)}{P(H)} & \text{otherwise} \end{cases}$$

certainty factor

$$CF = \frac{MB - MD}{1 - min(MB, MD)}$$

CF ranges between -1 (denial of H) and 1 (confirmation of H)

combining antecedent evidence

use of premises with less than absolute confidence

$$\begin{array}{ll} E_1 \wedge E_2 & min(CF(H,E_1), CF(H,E_1)) \\ E_1 \vee E_2 & min(CF(H,E_1), CF(H,E_1)) \\ \neg E & -CF(H,E) \end{array}$$

combining certainty factors for the same conclusion

several rules can be used to come to the same conclusion
applied incrementally as new evidence becomes available

$$CF_{rev}(CF_{old}, CF_{new}) =$$

$$\begin{cases} CF_{old} + CF_{new}(1 - CF_{old}) & both > 0 \\ CF_{old} + CF_{new}(1 + CF_{old}) & both < 0 \\ \frac{CF_{old} + CF_{new}}{1 - min(|CF_{old}|, |CF_{new}|)} & one < 0 \end{cases}$$

Advantages and Problems

+ simple implementation
+ better modeling of human experts' beliefs expression of belief and disbelief
+ successful application for certain problem classes
+ easier to gather than other values (no statistical base required
- (partially) ad hoc approach
- combination of non-independent evidence unsatisfactory
- new knowledge may require changes in the CFs of existing knowledge
- certainty factors can become the opposite of conditional probabilities for certain cases
- not suitable for long inference chains

Dempster-Shafer Theory

mathematical theory of evidence

frame of discernment FD
power set of the set of possible conclusions

mass probability function m
assigns a value from [0,1] to every item in the frame of discernment

mass probability $m(A)$
portion of the total mass probability that is assigned to an element A of FD
it cannot be further subdivided

belief $Bel(A)$ in a subset A
sum of the mass probabilities of all the proper subsets of A; likelihood that one of its members is the conclusion

certainty $Cer(A)$
interval $[Bel(A) \quad Pl(A)]$ range of belief
$Pl(A)$ is the plausibility, or maximum belief

combination of mass probabilities

$$m_1 \oplus m_2(C) = \frac{\sum_{X \cap Y = C} m_1(X) * m_2(Y)}{1 - \sum_{X \cap Y = \emptyset} m_1(X) * m_2(Y)}$$

where X, Y are hypothesis subsets, and C is their intersection

Advantages and Problems

+ clear, rigorous foundation
+ able to express confidence ("certainty about certainty") through intervals
- non-intuitive determination of mass probability
- usability somewhat unclear
- high computational overhead
- counterintuitive results due to normalization

Fuzzy Logic

linguistic variable
natural language term to describe concepts with vague values

fuzzy set
the categorization of elements x_i into a set S is described through a membership function $\mu_S(x)$ that associates each element x_i with a degree of membership in S

possibility measure $Poss\{x \in S\}$
degree to which an individual element x is a potential member in a fuzzy set S
possibility refers to allowed values; probability expresses expected occurrences of events

multiple premises

$$Poss(A \wedge B) = min(Poss(A), Poss(B))$$
$$Poss(A \vee B) = max(Poss(A), Poss(B))$$

fuzzy inference

$$Poss(B|A) = min(1, (1 - Poss(A) + Poss(B)))$$

implication according to Max-Min inference; also Max-Product inference and others

Advantages and Problems

+ rather general theory of uncertainty
+ wide applicability, many applications
+ natural use of vague and imprecise concepts
- membership functions can be difficult to find
- problems with long inference chains

State of Inexact Reasoning

no single best method
not even agreement on the measurement criteria

relations between approaches
is Dempster-Shafer a generalization of classical probability, or the other way round?

fringe cases
Dempster-Shafer seems to have problems with beliefs close to zero

computation
number of computations grows exponentially with the size of the problem space for initial Dempster-Shafer approaches

applications
fuzzy logic and certainty factors are widely used

revitalization of probability theory
re-examination of its foundations an usability

Chapter Review

Inexact Reasoning

Introduction
heuristics, probability, approaches

Objectives
expressiveness, soundness, complexity

Sources of Uncertainty
data, expert knowledge, representation, inference

Basics of Probability Theory
sample space, probability, compound and conditional prob.

Bayesian Approaches
inverse probability, Bayes' rule

Certainty Factors
measure of belief / disbelief, combining certainty factor

Dempster-Shafer Theory of Evidence
frame of discernment, mass probability, belief, certainty

Fuzzy Logic
linguistic variable, fuzzy set, possibility measure, fuzzy inference

State of Inexact Reasoning