

Logics and Networks for Human Reasoning

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Abstract We propose to model human reasoning tasks using completed logic programs interpreted under the three-valued Łukasiewicz semantics. Given an appropriate immediate consequence operator, completed logic programs admit a least model, which can be computed by iterating the consequence operator. Reasoning is then performed with respect to the least model. The approach is realized in a connectionist setting.

Key words: Human Reasoning, Logic Programs, Connectionist Models

1 Introduction

It has been widely argued in the field of Cognitive Science that logic is inadequate for modelling human reasoning (see e.g. [3]). In this context, “logic” is meant to be classical logic and, indeed, classical logic fails to capture some well-documented forms of human reasoning. However, in the field of Artificial Intelligence many non-classical logics have been studied and widely used. These logics try to capture many assumptions or features that occur in commonsense reasoning like, for example, the closed world assumption or non-monotonicity.

Recently, in [19] Stenning and van Lambalgen have suggested that completed logic programs under the three-valued Fitting semantics [9] can adequately model many human reasoning tasks. In addition, they propose a connectionist realization of their approach.

While trying to understand Stenning and van Lambalgen’s approach we made the following observations: *(i)* Łukasiewicz semantics [17] seems to be better suited for the approach as the law of equivalence holds under this semantics, whereas it does not hold under Fitting semantics. *(ii)* [19] contains some (minor) errors. *(iii)* The immediate consequence operator introduced in [19] differs in a subtle way from the one in [9] and it turned out, that the latter is inadequate for human reasoning. *(iv)* The core method, a connectionist model generator for logic programs first presented in [12], can easily be adapted to handle Stenning and van Lambalgen’s approach.

The paper discusses these observation by presenting three-valued logics, logic programs, their completion semantics as well as their immediate consequence operators in Section 2, by specifying an algorithm for mapping Stenning and van Lambalgen’s immediate consequence operator onto a recurrent neural network

$\neg$		$\wedge$		$\vee$	$\rightarrow_K$	$\rightarrow_L$	$\leftrightarrow_K$	$\leftrightarrow_L$	$\leftrightarrow_C$
$\top$	$\perp$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$
$\perp$	$\top$	$\perp$	$\top$	$\top$	$\top$	$\top$	$\perp$	$\perp$	$\perp$
$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	$\top$	$\top$	$\top$	$\top$	$\mathbf{U}$	$\mathbf{U}$	$\perp$
		$\top$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
		$\perp$	$\perp$	$\perp$	$\top$	$\top$	$\top$	$\top$	$\top$
		$\mathbf{U}$	$\perp$	$\perp$	$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	$\perp$
		$\top$	$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	$\perp$
		$\perp$	$\mathbf{U}$	$\perp$	$\mathbf{U}$	$\top$	$\mathbf{U}$	$\mathbf{U}$	$\perp$
		$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	$\mathbf{U}$	$\top$	$\mathbf{U}$	$\top$	$\top$

Table 1. Truth tables for 3-valued logics.

with feed-forward core in Section 3, by showing how some human reasoning task can be adequately modelled in the proposed logic and its connectionist realization in Section 4, and by discussing our findings in Section 5.

2 Logics, Programs and Consequence Operators

2.1 Three-Valued Logics

In this paper we consider (propositional logic) languages over an alphabet consisting of (propositional) variables, the connectives $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$ and parenthesis. We will consider various three-valued logics based on the semantics of their connectives (see Table 1): Łukasiewicz has proposed $\{\neg, \wedge, \vee, \rightarrow_L, \leftrightarrow_L\}$ [17], Kleene uses $\{\neg, \wedge, \vee, \rightarrow_K, \leftrightarrow_K\}$ in his *strong three-valued logic* [16] and $\{\neg, \wedge, \vee, \rightarrow_C, \leftrightarrow_C\}$ is suggested by Fitting for logic programming [9] and used by Stenning and van Lambalgen to model human reasoning [19].

An *interpretation* is a mapping from the language to the set of truth values $\{\top, \perp, \mathbf{U}\}$. Using Table 1 an interpretation for a given formula is uniquely determined by specifying the values for the propositional variables occurring in it. We will represent interpretations by pairs $\langle I^\top, I^\perp \rangle$, where $I^\top \cap I^\perp = \emptyset$, $I^\top$ contains all variables which are mapped to $\top$, $I^\perp$ contains all variables mapped to $\perp$, and all variables which occur neither in $I^\top$ nor in $I^\perp$ are mapped to $\mathbf{U}$. We use I_F and I_L to denote that an interpretation I uses Fitting or Łukasiewicz semantics, respectively. Furthermore, let $\mathcal{I}$ denote the set of all interpretation. $(\mathcal{I}, \subseteq)$ is a complete semilattice (see [9]). Finally, an interpretation I is said to be a *model* for a formula G iff $I(G) = \top$.

One should observe, that the law of equivalence (for all interpretations I : $I(F \leftrightarrow G) = I((F \rightarrow G) \wedge (G \rightarrow F))$) holds under Łukasiewicz semantics, but not under Fitting semantics.

2.2 Programs

A (*program*) *clause* is an expression of the form $A \leftarrow B_1 \wedge \dots \wedge B_n$, $n \geq 1$, where A is an atom and each B_i , $1 \leq i \leq n$, is either a literal (i.e., atom or negated

atom), $\top$ or $\perp$. A is called *head* and $B_1 \wedge \dots \wedge B_n$ *body* of the program clause. $\top$ is a valid formula, whereas $\perp$ is an unsatisfiable one. One should observe that the body of each clause is non-empty. A clause of the form $A \leftarrow \top$ is called *positive fact*. A clause of the form $A \leftarrow \perp$ is called *negative fact*. A (*logic*) *program* is a finite set of clauses. Two examples are $\mathcal{P}_1 = \{p \leftarrow q\}$ and $\mathcal{P}_2 = \{p \leftarrow q, q \leftarrow \perp\}$.

2.3 Completion

Let $\mathcal{P}$ be a program. It is turned into a single formula in the following way:

1. All clauses with the same head $A \leftarrow Body_1, \dots, A \leftarrow Body_n$ are replaced by the single formula $A \leftarrow Body_1 \vee \dots \vee Body_n$.
2. The resulting set is replaced by its conjunction.
3. If A is a variable occurring in $\mathcal{P}$ with no clause in $\mathcal{P}$ of the form $A \leftarrow Body$, then add $A \leftarrow \perp$.
4. All occurrences of $\leftarrow$ are replaced by $\leftrightarrow$.

The resulting formula is called *completion of $\mathcal{P}$* and is denoted by $comp(\mathcal{P})$. If the third step is omitted, then the resulting formula is called *weak completion of $\mathcal{P}$* and is denoted by $wcomp(\mathcal{P})$. For example, $wcomp(\mathcal{P}_1) = (p \leftrightarrow q) \neq comp(\mathcal{P}_1) = (p \leftrightarrow q) \wedge (q \leftrightarrow \perp) = comp(\mathcal{P}_2) = wcomp(\mathcal{P}_2)$.

Let $I = \langle \{p, q\}, \emptyset \rangle$. Then, $I_F(\mathcal{P}_2) = I_L(\mathcal{P}_2) = \top$, whereas $I_F(comp(\mathcal{P}_2)) = I_L(comp(\mathcal{P}_2)) = \perp$. The completion forces all models to map q to $\perp$, and the same holds for the weak completion.

Let $I = \langle \emptyset, \emptyset \rangle$. Then, $I_F(p \leftrightarrow p) = \top$, whereas $I_F(p \leftarrow p) = \perp$. In other words, I is a model for the completed program $p \leftrightarrow p$, but it is not a model for $p \leftarrow p$ under the Fitting semantics. On the other hand, $I_L(p \leftrightarrow p) = I_L(p \leftarrow p) = \top$.

2.4 Consequence Operators

In [9] Fitting has defined an immediate consequence operator $\Phi_{F,\mathcal{P}}(I) = \langle J^\top, J^\perp \rangle$, where $J^\top = \{A \mid A \leftarrow Body \in \mathcal{P} \text{ and } I(Body) = \top\}$ and $J^\perp = \{A \mid \text{for all } A \leftarrow Body \in \mathcal{P} : I(Body) = \perp\}$. One should note that $I_F(Body) = I_L(Body)$ because the body of a clause is a conjunction of literals. Fitting has shown that $\Phi_{F,\mathcal{P}}$ is monotone on $(\mathcal{I}, \subseteq)$.

Proposition 1. $\Phi_{F,\mathcal{P}}$ is continuous and admits a least fixed point $lfp(\Phi_{F,\mathcal{P}})$.

Proof Because our programs are finite and, thus, the underlying alphabet and all directed subsets of $\mathcal{I}$ are finite, we find that a monotone $\Phi_{F,\mathcal{P}}$ is also continuous (see e.g. [20]). Hence, $\Phi_{F,\mathcal{P}}$ admits a least fixed point, which can be computed by iterating $\Phi_{F,\mathcal{P}}$ starting with the empty interpretation. $\square$

$lfp(\Phi_{F,\mathcal{P}})$ is equal to the least model of $comp(\mathcal{P})$ under Fitting semantics. For example, $lfp(\Phi_{F,\mathcal{P}_1}) = lfp(\Phi_{F,\mathcal{P}_2}) = \langle \emptyset, \{p, q\} \rangle$.

In [19] Stenning and van Lambalgen have defined a slightly different immediate consequence operator: $\Phi_{svL,\mathcal{P}}(I) = \langle J^\top, J^\perp \rangle$, where $J^\top = \{A \mid A \leftarrow Body \in \mathcal{P} \text{ and } I(Body) = \top\}$ and $J^\perp = \{A \mid \text{there exists } A \leftarrow Body \in \mathcal{P} \text{ and for all } A \leftarrow Body \in \mathcal{P} : I(Body) = \perp\}$. They showed that this operator is also monotone.

Proposition 2. $\Phi_{SvL, \mathcal{P}}$ is continuous and admits a least fixed point $lfp(\Phi_{SvL, \mathcal{P}})$.

This result can be proven along the lines of the proof of Propostion 1. For example, $lfp(\Phi_{SvL, \mathcal{P}_1}) = \langle \emptyset, \emptyset \rangle$ and $lfp(\Phi_{SvL, \mathcal{P}_2}) = \langle \emptyset, \{p, q\} \rangle$. In addition, Stenning and van Lambalgen make the following claims, where they assume that models are defined with respect to Fitting semantics:

- A. The least fixed point of $\Phi_{SvL, \mathcal{P}}$ can be shown to be the minimal model of $\mathcal{P}$ (Lemma 4(1.) in [19]).
- B. All models of $comp(\mathcal{P})$ are fixed points of $\Phi_{SvL, \mathcal{P}}$ (Lemma 4(3.) in [19]).

Both claims are false. Consider $\mathcal{P}_1 = \{p \leftarrow q\}$ and let $I = \langle \emptyset, \emptyset \rangle$. As discussed before, $lfp(\Phi_{SvL, \mathcal{P}_1}) = I$, but $I_F(\mathcal{P}_1) = \mathbb{U}$; thus, we have obtained a counter example for A. One should observe that the minimal models of $\mathcal{P}_1$ under Fitting semantics are $\langle \{p\}, \emptyset \rangle$ and $\langle \emptyset, \{q\} \rangle$, both of which are not fixed points of $\Phi_{SvL, \mathcal{P}_1}$. Now let $I'' = \langle \emptyset, \{p, q\} \rangle$ and $I' = \langle \emptyset, \{p\} \rangle$. $I_F''(comp(\mathcal{P}_1)) = \top$, but $\Phi_{SvL, \mathcal{P}_1}(I'') = I'$, $\Phi_{SvL, \mathcal{P}_1}(I') = I$, and $\Phi_{SvL, \mathcal{P}_1}(I) = I$; thus, we have obtained a counter example for B.

Problem A. can be overcome if we interpret programs under Lukasiewicz semantics. For the discussed example we find $I_L(\mathcal{P}_1) = \top$, which is no coincidence as we will show in the sequel.

Proposition 3. (i) If $I_L(wcomp(\mathcal{P})) = \top$ then $\Phi_{SvL, \mathcal{P}}(I) \subseteq I$.
(ii) If $\Phi_{SvL, \mathcal{P}}(I) = I$ then $I_L(wcomp(\mathcal{P})) = \top$.

Proof Sketch (i) If $I_L(wcomp(\mathcal{P})) = \top$ then for each equivalence $A \leftrightarrow Body_1 \vee \dots \vee Body_n$ occurring in $wcomp(\mathcal{P})$ we find that

$$I_L(A) = I_L(Body_1 \vee \dots \vee Body_n). \quad (1)$$

Now let $I = \langle I^\top, I^\perp \rangle$ and $\langle J^\top, J^\perp \rangle = \Phi_{SvL, \mathcal{P}}(\langle I^\top, I^\perp \rangle)$. By definition of $\Phi_{SvL, \mathcal{P}}$ we find $J^\top \subseteq I^\top$ and $J^\perp \subseteq I^\perp$ given (1). Hence, $\Phi_{SvL, \mathcal{P}}(I) \subseteq I$.

(ii) Suppose $\Phi_{SvL, \mathcal{P}}(I) = I$. Let $F := A \leftrightarrow Body_1 \vee \dots \vee Body_n$ be an arbitrary but fixed conjunct occurring in $wcomp(\mathcal{P})$. If $I_L(A) = \top$, then there exists $A \leftarrow Body_i \in \mathcal{P}$ such that $I_L(Body_i) = \top$. Hence, $I_L(Body_1 \vee \dots \vee Body_n) = \top$ and, consequently, $I_L(F) = \top$. The cases $I_L(A) = \perp$ and $I_L(A) = \mathbb{U}$ follow similarly. Because F was arbitrary but fixed we conclude $I_L(wcomp(\mathcal{P})) = \top$. $\square$

Proposition 4. If $I = lfp(\Phi_{SvL, \mathcal{P}})$ then $I_L(wcomp(\mathcal{P})) = I_L(\mathcal{P}) = \top$.

Proof Sketch From Proposition 3(ii) we learn that $I = lfp(\Phi_{SvL, \mathcal{P}})$ entails $I_L(wcomp(\mathcal{P})) = \top$. Moreover, $I_L(wcomp(\mathcal{P})) = \top$ iff for each equivalence $A \leftrightarrow Body_1 \vee \dots \vee Body_n$ occurring in $wcomp(\mathcal{P})$ we find that $I_L(A) = I_L(Body_1 \vee \dots \vee Body_n)$. A careful case analysis reveals that $I_L(\mathcal{P}) = \top$ holds. $\square$

3 The Core Method

In [12] a connectionist model generator for propositional logic programs using recurrent networks with feed-forward core was presented. It was later called the *core method* [2]. The core method has been extended and applied to a variety of programs including modal (see e.g. [7]) and first-order logic programs [1]. It is based on the idea that feed-forward connectionist networks can approximate almost all functions arbitrarily well [14,11] and, hence, they can also approximate – and in some cases compute – the immediate consequence operators associated with logic programs. Moreover, if such an operator is a contraction mapping on a complete metric space, then Banach’s contraction mapping theorem ensures that a unique fixpoint exists such that the sequence constructed from applying the operator iteratively to any element of the metric space converges to the fixed point [10]. Turning the feed-forward core into a recurrent network allows to compute or approximate the least model of a logic program [13].

Kalinke has applied the core method to logic programs under the Fitting semantics presented in Section 2 [15]. In particular, her feed-forward cores compute $\Phi_{F,\mathcal{P}}$ for any given program $\mathcal{P}$. Seda and Lane showed that the core method can be extended to many-valued logic programs [18]. Restricted to three-valued logic programs considered here, their cores also compute $\Phi_{F,\mathcal{P}}$. In the sequel, these approaches are modified in order to compute $\Phi_{SvL,\mathcal{P}}$.

Given a program $\mathcal{P}$, the following algorithm translates $\mathcal{P}$ into a feed-forward core. Let m be the number of propositional variables occurring in $\mathcal{P}$. Without loss of generality, we may assume that the variables are denoted by natural numbers from $[1, m]$. Let $\omega \in \mathbb{R}^+$.

1. The input and output layer is a vector of binary threshold units of length $2m$ representing interpretations. The $2i - 1$ -st unit in the layers, denoted by $i^\top$, is active iff the i -th variable is mapped to $\top$. The $2i$ -th unit in the layers, denoted by $i^\perp$, is active iff the i -th variable is mapped to $\perp$. Both, the $2i - 1$ -st and the $2i$ -th unit, are passive iff the i -th variable is mapped to U . The case where the $2i - 1$ -st and the $2i$ -th unit are active is not allowed. The threshold of each unit occurring in the input layer is set to $\frac{1}{2}$. The threshold of each $2i - 1$ -st unit occurring in the output layer is set to $\frac{\omega}{2}$. The threshold of each $2i$ -th unit occurring in the output layer is set to $\max\{\frac{\omega}{2}, l - \frac{\omega}{2}\}$, where l is the number of clauses with head i in $\mathcal{P}$. In addition, two units representing $\top$ and $\perp$ are added to the input layer. The threshold of these units is set to $-\frac{1}{2}$.
2. For each clause of the form $A \leftarrow B_1 \wedge \dots \wedge B_k$ occurring in $\mathcal{P}$, do the following.
 - (a) Add two binary threshold units $h^\top$ and $h^\perp$ to the hidden layer.
 - (b) Connect $h^\top$ to the unit $A^\top$ in the output layer. Connect $h^\perp$ to the unit $A^\perp$ in the output layer.
 - (c) For each B_j , $1 \leq j \leq k$, do the following.
 - i. If B_j is an atom, then connect the units $B_j^\top$ and $B_j^\perp$ in the input layer to $h^\top$ and $h^\perp$, respectively.

- ii. If B_j is the literal $\neg B$, then connect the units $B^\perp$ and $B^\top$ in the input layer to $h^\top$ and $h^\perp$, respectively.
 - iii. If B_j is $\top$, then connect the unit $\top$ in the input layer to $h^\top$.
 - iv. If B_j is $\perp$, then connect the unit $\perp$ in the input layer to $h^\perp$.
- (d) Set the threshold of $h^\top$ to $k - \frac{\omega}{2}$, and the threshold of $h^\perp$ to $\frac{\omega}{2}$.
3. Set the weights associated with all connections to ω .

Proposition 5. *For each program $\mathcal{P}$, there exists a core of binary threshold units computing $\Phi_{SvL, \mathcal{P}}$.*

Proof Assume that the input layer is activated at time t such that it represents an interpretation I . Then, at time $t + 1$ an $h^\top$ -unit representing $A \leftarrow B_1 \wedge \dots \wedge B_k$ in the hidden layer becomes active iff all units representing $B_1, \dots, B_k$ in the input layer are active, i.e., if $I(B_1) = \dots = I(B_k) = \top$. Likewise, at time $t + 1$ an $h^\perp$ -unit representing $A \leftarrow B_1 \wedge \dots \wedge B_k$ in the hidden layer becomes active iff one unit representing the negation of $B_1, \dots, B_k$ in the input layer is active, i.e., if $I(\neg B_1) \vee \dots \vee I(\neg B_k) = \top$. At time $t + 2$ a unit representing A in the output layer becomes active iff there is an active $h^\top$ -unit representing $A \leftarrow B_1 \wedge \dots \wedge B_k$ at time $t + 1$. Likewise, at time $t + 2$ a unit representing $\neg A$ in the output layer becomes active iff all $h^\perp$ units representing rules with head A are active at time $t + 1$. Thus, the core is a direct encoding of $\Phi_{SvL, \mathcal{P}}$. $\square$

Given a program $\mathcal{P}$ and its core, a recurrent network can be constructed by connecting each unit in the output layer to its corresponding unit in the input layer with weight 1. In Figure 1 the construction is illustrated.

Proposition 6. *For each program $\mathcal{P}$, the corresponding recurrent network initialized by the empty interpretation will converge to a stable state which corresponds to the least fixed point of $\Phi_{SvL, \mathcal{P}}$.*

Proof The result follows immediately from the construction of the recurrent network using Propositions 4 and 5. $\square$

4 Human Reasoning

In this section we will discuss some examples taken from [3]. These examples were used by Byrne to show that classical logic cannot appropriately model human reasoning. Stenning and van Lambalgen argue that a three-valued logic programs under a completion semantics can well model human reasoning [19]. Moreover, as we will see, the core method presented in Section 3 serves as a connectionist model generator in these cases.

Consider the following sentences: *If Marian has an essay to write, she will study late in the library. She has an essay to write.* In [3] 96% of all subjects conclude that *Marian will study late in the library*. The two sentences can be represented by the program $\mathcal{P}_3 = \{l \leftarrow e \wedge \neg ab, e \leftarrow \top, ab \leftarrow \perp\}$. The first

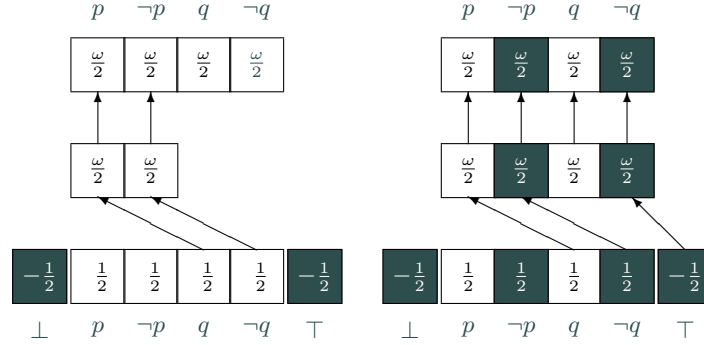


Figure 1. The stable states of the feed-forward cores for $\mathcal{P}_1$ (left) and $\mathcal{P}_2$ (right), where all connections have weight ω , active units are shown in grey and passive units in white. The recurrent connections between corresponding units in the output and input layer are not shown.

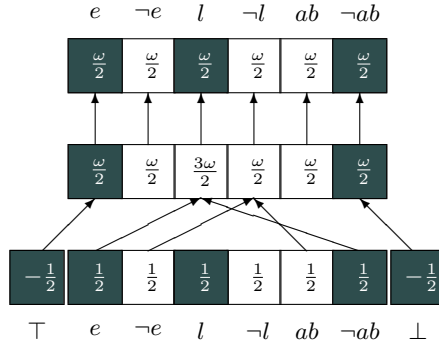


Figure 2. The stable state of the feed-forward core for $\mathcal{P}_3$.

sentence is interpreted as a licence for a conditional and the atom ab is used to cover all additional preconditions that we may be unaware of. As we know of no such preconditions, the rule $ab \leftarrow \perp$ is added. The corresponding network as well as its stable state are shown in Figure 2. From $\text{lfp}(\Phi_{SvL, \mathcal{P}_3}) = \langle \{l, e\}, \{ab\} \rangle$ follows that Marian will study late in the library.

Suppose now that the antecedent is denied: *If Marian has an essay to write, she will study late in the library. She does not have an essay to write.* In [3] 46% of subjects conclude that Marian will not study late in the library. These subject err with respect to classical logic. But they do not err with respect to the non-classical logic considered here. The two sentences can be represented by the program $\mathcal{P}_4 = \{l \leftarrow e \wedge \neg ab, e \leftarrow \perp, ab \leftarrow \perp\}$. The corresponding network as well as its stable state are shown in Figure 3. From $\text{lfp}(\Phi_{SvL, \mathcal{P}_4}) = \langle \emptyset, \{ab, e, l\} \rangle$ follows that Marian will not study late in the library.

Now consider an alternative argument: *If Marian has an essay to write, she will study late in the library. She does not have an essay to write. If she*

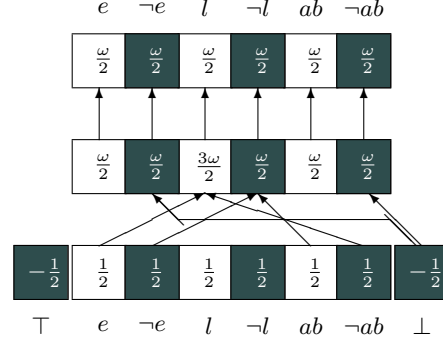


Figure 3. The stable state of feed-forward core for $\mathcal{P}_4$.

has textbooks to read, she will study late in the library. In [3] 4% of subjects conclude that Marian will not study late in the library. These sentences can be represented by $\mathcal{P}_5 = \{l \leftarrow e \wedge \neg ab_1, e \leftarrow \perp, ab_1 \leftarrow \perp, l \leftarrow t \wedge \neg ab_2, ab_2 \leftarrow \perp\}$. Due to lack of space we leave the construction of the network to the interested reader. From $\text{lfp}(\Phi_{SvL, \mathcal{P}_5}) = \langle \emptyset, \{ab_1, ab_2, e\} \rangle$ follows that it is unknown whether Marian will study late in the library. One should observe that $\text{lfp}(\Phi_{F, \mathcal{P}_5}) = \langle \emptyset, \{ab_1, ab_2, e, t, l\} \rangle$ and, consequently, one would conclude that Marian will not study late in the library. Thus, Fitting's operator leads to a wrong answer with respect to human reasoning, whereas Stenning and van Lambalgen's operator does not.

As final example consider the presence of an additional argument: *If Marian has an essay to write, she will study late in the library. She has an essay to write. If the library stays open, she will study late in the library.* In [3] 38% of subjects conclude that Marian will study late in the library. These sentences can be represented by $\mathcal{P}_6 = \{l \leftarrow e \wedge \neg ab_1, e \leftarrow \top, l \leftarrow o \wedge \neg ab_2, ab_1 \leftarrow \neg o, ab_2 \leftarrow \neg e\}$. As argued in [19] the third sentence gives rise to an additional argument for studying in the library, viz. that the library is open. Likewise, there must be a reason for going to the library like, for example, writing an essay. The corresponding network as well as its stable state are shown in Figure 4. From $\text{lfp}(\Phi_{SvL, \mathcal{P}_6}) = \langle \{e\}, \{ab_2\} \rangle$ follows that it is unknown whether Marian will study late in the library.

5 Discussion

We propose to use the Lukasiewicz semantics for three-valued logic programs in the area of human reasoning. Returning to our last example, $\text{lfp}(\Phi_{SvL, \mathcal{P}_6}) = \langle \{e\}, \{ab_2\} \rangle$ is a model for both, the weak completion of $\mathcal{P}_6$ and $\mathcal{P}_6$ itself, under the Lukasiewicz semantics, whereas it is – somewhat surprisingly – a model for the weak completion of $\mathcal{P}_6$ but not for $\mathcal{P}_6$ under the Fitting semantics. Under the Lukasiewicz semantics the completion of a three-valued logic program is exactly what completion was originally thought of, viz., the addition of the only-if halves

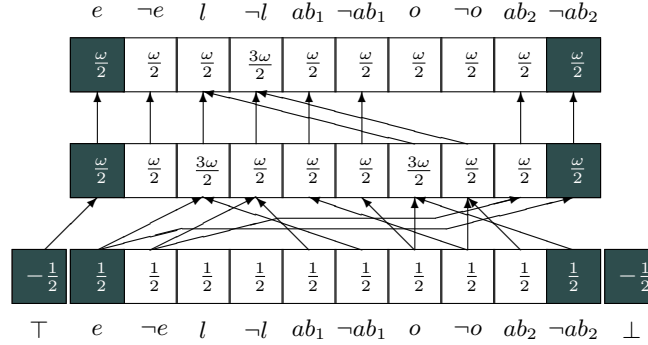


Figure 4. The stable state of feed-forward core for $\mathcal{P}_6$.

to a program specifying the if-halves [4]. Nevertheless, whether Łukasiewicz semantics is adequate for human reasoning in a broader sense remains to be seen.

We showed that the core method can be adapted to implement the revised immediate consequence operator of Stenning and van Lambalgen. We presented an algorithm for constructing the networks and proved that the networks settle down in a state encoding the least fixed point of the operator. Although our networks consist of logical threshold units, they can be replaced by bipolar sigmoidal ones while preserving the relationship to logic programs by applying the method first presented in [8] (see also [5]). The modified networks can then be trained using backpropagation or related techniques, rule extraction methods can be applied to the trained networks and the neural-symbolic cycle can be closed. On the other hand, to the best of our knowledge there is no evidence that backpropagation is neurally plausible.

In our networks units come in pairs, where the first (second) element represents the fact that the corresponding variable or formula is mapped to true (false). In [19] similar networks are proposed – albeit in a three-dimensional setting – and, in addition, the elements of each pair inhibit each other. From a logical point of view such an inhibition is unnecessary – it can never be the case that both elements are active – as long as the units of the input layer are not externally activated. In a general setting, however, where context information is used to activate the input units, such inhibitory connections establishing a winner-take-all behaviour are very useful. Unfortunately, the presented results concerning the core networks and their relation to logic programs do not apply in this case anymore unless we would be able to show that $\Phi_{SvL, \mathcal{P}}$ is a contraction.

In [19] integrity constraints and abduction are suggested to handle additional human reasoning tasks. We would like to investigate whether the techniques developed in [6] can be applied to model the tasks in a connectionist setting.

In the field of Logic Programming Fitting’s three-valued (first-order) logic has been used in termination analysis. It remains to be seen whether these results carry over to Łukasiewicz semantics. How important is Stenning and van Lambalgen’s operator from a logic programming perspective?

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