

Exercises

1. Let M be the deterministic finite automaton defined by

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b\}$$

$$F = \{q_2\}$$

δ	a	b
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_2	q_0

- Give the state diagram of M .
- Trace the computations of M that process the strings $abaa$, $bbbabb$, $bababa$, and $bbbaa$.
- Which of the strings from part (b) are accepted by M ?
- Give a regular expression for $L(M)$.

✓ 2. Let M be the deterministic finite automaton

$$Q = \{q_0, q_1, q_2\}$$

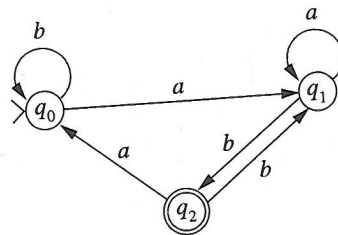
$$\Sigma = \{a, b\}$$

$$F = \{q_0\}$$

δ	a	b
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_0

- Give the state diagram of M .
- Trace the computation of M that processes $babaab$.
- Give a regular expression for $L(M)$.
- Give a regular expression for the language accepted if both q_0 and q_1 are accepting states.

✓ 3. Let M be the DFA with state diagram



- Construct the transition table of M .
- Which of the strings $baba$, $baab$, $abab$, $abaaab$ are accepted by M ?
- Give a regular expression for $L(M)$.

- *4. The recursive step in the definition of the extended transition function (Definition 5.2.4) may be replaced by $\hat{\delta}'(q_i, au) = \hat{\delta}'(\delta(q_i, a), u)$, for all $u \in \Sigma^*$, $a \in \Sigma$, and $q_i \in Q$. Prove that $\hat{\delta} = \hat{\delta}'$.

For Exercises 5 through 21, build a DFA that accepts the described language.

- ✓ 5. The set of strings over $\{a, b, c\}$ in which all the a 's precede the b 's, which in turn precede the c 's. It is possible that there are no a 's, b 's, or c 's.
- ✓ 6. The set of strings over $\{a, b\}$ in which the substring aa occurs at least twice.
- ✓ 7. The set of strings over $\{a, b\}$ that do not begin with the substring aaa .
- ✓ 8. The set of strings over $\{a, b\}$ that do not contain the substring aaa .
- ✓ 9. The set of strings over $\{a, b, c\}$ that begin with a , contain exactly two b 's, and end with cc .
- ✓ 10. The set of strings over $\{a, b, c\}$ in which every b is immediately followed by at least one c .
- ✓ 11. The set of strings over $\{a, b\}$ in which the number of a 's is divisible by three.
12. The set of strings over $\{a, b\}$ in which every a is either immediately preceded or immediately followed by b , for example, $baab$, aba , and b .
13. The set of strings of odd length over $\{a, b\}$ that contain the substring bb .
14. The set of strings over $\{a, b\}$ that have odd length or end with aaa .
15. The set of strings of even length over $\{a, b, c\}$ that contain exactly one a .
16. The set of strings over $\{a, b\}$ that have an odd number of occurrences of the substring aa . Note that aaa has two occurrences of aa .
17. The set of strings over $\{a, b\}$ that contain an even number of substrings ba .
18. The set of strings over $\{1, 2, 3\}$ the sum of whose elements is divisible by six.
19. The set of strings over $\{a, b, c\}$ in which the number of a 's plus the number of b 's plus twice the number of c 's is divisible by six.
20. The set of strings over $\{a, b\}$ in which every substring of length four has at least one b . Note that every substring with length less than four is in this language.
- * 21. The set of strings over $\{a, b, c\}$ in which every substring of length four has exactly one b .
- ✓ 22. For each of the following languages, give the state diagram of a DFA that accepts the languages.

✓ a) $(ab)^*ba$

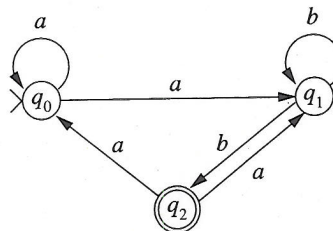
✓ b) $(ab)^*(ba)^*$

✓ c) $aa(a \cup b)^+bb$

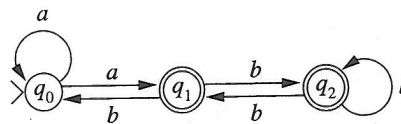
✓ d) $((aa)^+bb)^*$

✓ e) $(ab^*a)^*$

✓ 23. Let M be the nondeterministic finite automaton



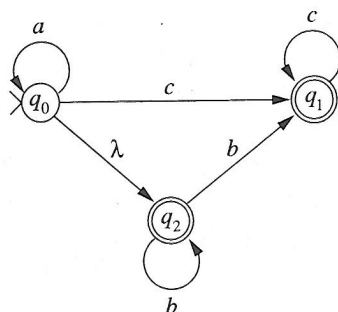
- ✓ a) Construct the transition table of M .
 - ✓ b) Trace all computations of the string $aaabb$ in M .
 - ✓ c) Is $aaabb$ in $L(M)$?
 - d) Give a regular expression for $L(M)$.
24. Let M be the nondeterministic finite automaton



- a) Construct the transition table of M .
 - b) Trace all computations of the string $aabb$ in M .
 - c) Is $aabb$ in $L(M)$?
 - d) Give a regular expression for $L(M)$.
 - e) Construct a DFA that accepts $L(M)$.
 - f) Give a regular expression for the language accepted if both q_0 and q_1 are accepting states.
25. For each of the following languages, give the state diagram of an NFA that accepts the languages.
- a) $(a \cup ab \cup aab)^*$
 - ✓ b) $(ab)^* \cup a^*$
 - ✓ c) $(abc)^*a^*$
 - ✓ d) $(ba \cup bb)^* \cup (ab \cup aa)^*$
 - ✓ e) $(ab^+a)^+$
26. Give a recursive definition of the extended transition function $\hat{\delta}$ of an NFA- λ . The value $\hat{\delta}(q_i, w)$ is the set of states that can be reached by computations that begin at node q_i and completely process the string w .

For Exercises 27 through 34, give the state diagram of an NFA that accepts the given language. Remember that an NFA may be deterministic, but you should use nondeterminism whenever it is appropriate.

27. The set of strings over $\{a, b\}$ that contain either aa and bb as substrings.
28. The set of strings over $\{a, b\}$ that contain both or neither aa and bb as substrings.
- *29. The set of strings over $\{a, b\}$ whose third-to-the-last symbol is b .
30. The set of strings over $\{a, b\}$ whose third and third-to-last symbols are both b . For example, $aababaa$, $abbbbbbb$, and $abba$ are in the language.
31. The set of strings over $\{a, b\}$ in which every a is followed by b or ab .
32. The set of strings over $\{a, b\}$ that have a substring of length four that begins and ends with the same symbol.
33. The set of strings over $\{a, b\}$ that contain substrings aaa and bbb .
34. The set of strings over $\{a, b, c\}$ that have a substring of length three containing each of the symbols exactly once.
35. Construct the state diagram of a DFA that accepts the strings over $\{a, b\}$ ending with the substring $abba$. Give the state diagram of an NFA with six arcs that accepts the same language.
36. Let M be the NFA- λ



- a) Compute λ -closure(q_i) for $i = 0, 1, 2$.
- b) Give the input transition function t for M .
- c) Use Algorithm 5.6.3 to construct a state diagram of a DFA that is equivalent to M .
- d) Give a regular expression for $L(M)$.