Exercises

1. Let $M$ be the PDA defined by

$$
Q = \{q_0, q_1, q_2\} \quad \delta(q_0, a, \lambda) = \{(q_0, A)\}
$$

$$
\Sigma = \{a, b\} \quad \delta(q_0, \lambda, \lambda) = \{(q_1, \lambda)\}
$$

$$
\Gamma = \{A\} \quad \delta(q_0, b, A) = \{(q_2, \lambda)\}
$$

$$
F = \{q_1, q_2\} \quad \delta(q_1, \lambda, A) = \{(q_1, \lambda)\}
$$

$$
\delta(q_2, b, A) = \{(q_2, \lambda)\}
$$

$$
\delta(q_2, \lambda, A) = \{(q_2, \lambda)\}.
$$

a) Describe the language accepted by $M$.

b) Give the state diagram of $M$.

c) Trace all computations of the strings $aab$, $abb$, $aba$ in $M$.

d) Show that $aab$, $aaab \in L(M)$.

2. Let $M$ be the PDA in Example 7.1.3.

a) Give the transition table of $M$.

b) Trace all computations of the strings $ab$, $abb$, $abbb$ in $M$.

c) Show that $aaaa$, $baab \in L(M)$.

d) Show that $aaa$, $ab \notin L(M)$.

3. Construct PDAs that accept each of the following languages.

a) $\{a^i b^j \mid 0 \leq i \leq j\}$

b) $\{a^i b^j c^k \mid i, j, k \geq 0\}$

c) $\{a^i b^j c^k \mid i + k = j\}$

d) $\{w \mid w \in \{a, b\}^* \text{ and } w \text{ has twice as many } a's \text{ as } b's\}$

e) $\{a^i b^j c^k \mid i + j = k\}$

f) $\{a^i b^j \mid i \geq 0 \text{ or } a^* \cup b^*\}$

g) $\{a^i b^j \mid i = j\}$

h) $\{a^i b^j c^k \mid i = j \text{ or } j + k = i\}$

i) $\{a^i b^j c^k \mid i + j > 0\}$

j) The set of palindromes over $\{a, b\}$

4. Construct a PDA with only two stack elements that accepts the language

$$
\{w a^i b^j c^k w^R \mid w \in \{a, b, c\}^*\}.
$$
5. Give the state diagram of a PDA $M$ that accepts $\{a^i b^{i+j} \mid 0 \leq j \leq i\}$ with acceptance by empty stack. Explain the role of the stack symbols in the computation of $M$. Trace the computations of $M$ with input $aabb$ and $aaabb$.

6. The machine $M$

![State diagram of PDA](image)

accepts the language $L = \{a^i b^i \mid i > 0\}$ by final state and empty stack.

a) Give the state diagram of a PDA that accepts $L$ by empty stack.
b) Give the state diagram of a PDA that accepts $L$ by final state.

7. Let $L$ be the language $\{w \in \{a, b\}^* \mid w$ has a prefix containing more $b$'s than $a$'s\}. For example, $baa, abba, abbaaa \in L$, but $aabb, aabab \notin L$.

a) Construct a PDA that accepts $L$ by final state.
b) Construct a PDA that accepts $L$ by empty stack.

8. Let $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ be a PDA that accepts $L$ by final state and empty stack. Prove that there is a PDA that accepts $L$ by final state alone.

9. Let $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ be a PDA that accepts $L$ by final state and empty stack. Prove that there is a PDA that accepts $L$ by empty stack alone.

10. Let $L = \{a^i b^i \mid i \geq 0\}$.

\[ \sqrt{\sqrt{a}} \]

a) Construct a PDA $M_1$ with $L(M_1) = L$.
b) Construct an atomic PDA $M_2$ with $L(M_2) = L$.
c) Construct an extended PDA $M_3$ with $L(M_3) = L$ that has fewer transitions than $M_1$.
d) Trace the computation that accepts the string $aab$ in each of the automata constructed in parts (a), (b), and (c).

11. Let $L = \{a^i b^3 \mid i \geq 0\}$.

\[ \sqrt{\sqrt{a}} \]

a) Construct a PDA $M_1$ with $L(M_1) = L$.
b) Construct an atomic PDA $M_2$ with $L(M_2) = L$.
c) Construct an extended PDA $M_3$ with $L(M_3) = L$ that has fewer transitions than $M_1$.
d) Trace the computation that accepts the string $aabb$ in each of the automata constructed in parts (a), (b), and (c).

12. Use the technique of Theorem 7.3.1 to construct a PDA that accepts the language of the Greibach normal form grammar

\[
S \rightarrow aABA | aBB \\
A \rightarrow bA | b \\
B \rightarrow cB | c.
\]