Fill in the answers.

1. (1 point) Given the alphabet \( \Sigma = \{1, 22, 3\} \).
   Check every item in the following list that is a string over this alphabet
   (one wrong answer will cost you the point)
   
   \_V\_111, \_111222, \_312, \_V\_2222, \_1000, \_aaabb, \_V\_\lambda

2. (1.5 points) Given the alphabet \( \Sigma = \{a, b, c\} \).
   Check every item in the following list that is a language over this alphabet
   (one wrong answer will cost you the point)
   
   \_\lambda, \_V\_\{\lambda\}, \_V\_\emptyset, \_V\_\{\emptyset\}, \_a, \_V\_\{a\}, \_V\_\Sigma, \_V\_\Sigma^*

3. (1.5 points: 0.5 point for each) Given strings \( u \) and \( v \) over the alphabet \( \Sigma \).
   Is the following true: \((uv)^R = u^Rv^R\) ? (yes/no).\_\_no\_\_  
   Is the following true: \(uv = vu\) ?(yes/no).\_\_no\_\_  
   Is the following true: for any \( i>0 \) natural number \((uv)^i = u^iv^i\) ?(yes/no).\_\_no\_\_

4. (1 point) Given the alphabet \( \Sigma \).
   The set of all strings over \( \Sigma \) is (choose the correct answer)
   
   \_finite, \_V\_denumerable, \_uncountable

5. (1 point) Given the alphabet \( \Sigma \).
   The set of all languages over \( \Sigma \) is (choose the correct answer)
   
   \_finite, \_denumerable, \_uncountable

6. (1 point) Given the recursive definition of a language \( L \) over the alphabet \{a,b\}

   **Basis:** \( a \in L \)  
   **Recursive step:** if \( u \in L \) then \( au \in L \) and \( ub \in L \).  
   **Closure:** a string is in \( L \) if it can be obtained from the basic element by finite number of applications of the recursive step.

   Check every item in the following list that is a string of \( L \)
   (one wrong answer will cost you the point)
   
   \_V\_aaa, \_bbb, \_ababab, \_V\_abbb, \_abbbaba, \_V\_aaabb, \_\lambda
7. (1 point) Given the following regular expressions over the alphabet \{a, b\}

\[
1) \quad (a^* \cup b^*)^* \quad 2) \quad (ab)^* \quad 3) \quad (a \cup b)^*
\]

Which regular expressions are equivalent? Check the correct answer.

_____ 1) and 2) are equivalent, but they are not equivalent to 3).
_____ 1) and 3) are equivalent, but they are not equivalent to 2).
_____ 2) and 3) are equivalent, but they are not equivalent to 1).
_____ there are no equivalent regular expressions among those listed.
_____ all listed regular expressions are equivalent to each other.

8. (1 point) Given a language over the alphabet \{a, b\} defined with the help of a regular expression

\[ a^* b^* \cup (ab)^+ \]

Check all the strings that are strings of \(L\) (one wrong answer will cost you the point)

_____ bbb, ____ aaa, __ V aaabb, _ V ababab, ____ aabbab, __ V abbbbb, ___ \(\lambda\)

9. (1 point) Given set \(X = \{a, b\}\).

How many elements does the set \(X^5\) have? (give a number) \(2^5\) \(=\) all 5-length strings

10. (1 point) Let \(X = \{aa, bb, cc\}\) and \(Y = \{a, b\}\).

How many elements does the set \(XY\) have? (give a number) \(6\) .

11. (1 point) Let \(X = \emptyset\).

How many elements does the set \(X^*\) have? (give a number) \(1\) \(=\) \(\emptyset^* = \{\lambda\}\)

12. (1 point) Given a set \(X\). How can you obtain the set \(X^*\) (using set operations), if the set \(X^+\) is given?

\[ X^* = X^+ \cup \{\lambda\} \]

13. (1 point) Given languages \(X = \{a, aa, b\}\), \(Y = \{a^i \mid i > 0\}\) over the alphabet \(\Sigma = \{a, b\}\). Is the language \(L = X \cap Y\) a regular set over \(\Sigma\)? (yes/no) \(\text{yes}\)

\(\text{// } X \cap Y = \{a, aa\} = \{a\} \cup \{a\} \{a\}\)

14. (1 point) Given alphabet \(\Sigma = \{a, b\}\). Is \(a(a \cup b)^*a \cap (a \cup b)^*(a \cup b)^*\) a regular expression over the alphabet \(\Sigma\)? (yes/no) \(\text{no}\) \(\text{// it contains an illegal symbol } \cap\)

15. (1 point) List the basic regular sets over a given alphabet \(\Sigma\) (the sets mentioned in the basis of the recursive definition of the regular sets).

\(\emptyset, \{\lambda\} \text{ and } \{a\} \text{ for every } a \in \Sigma\)

16. (1 point) List the set operations that are used in the recursive step of the recursive definition of the regular sets over a given alphabet \(\Sigma\) (the set operations that are used to obtain new regular sets from the “old” ones).

\(\text{union, concatenation, Kleene star}\)