Picture 1. Diagram of a DFA \( M = (Q, \Sigma, \delta, q_0, F) \)

1. (3 points: 0.5 point each) Given the DFA \( M = (Q, \Sigma, \delta, q_0, F) \) as defined in the picture 1.
   a) Define the following components of the mathematical system \( M \)
      (no mistakes for the credit).
      \[
      Q = \{ q_0, q_1, q_2, q_3 \} \quad \Sigma = \{ a, b, c \} \quad F = \{ q_2 \}
      \]
   b) Define the value of the transition function \( \delta \) for the pair \([q_1, a]\) : \( \delta(q_1, a) = \quad q_3 \quad \)
   c) Given a string \( w \in \Sigma^* \). The DFA \( M \) is processing \( w \) and finishing the work in the \( q_3 \) state rejecting \( w \). Give the starting instantaneous machine configuration of \( M \) (the configuration specifying \( M \) at the moment when the work starts) \( [q_0, w] \)
   d) Give the ending instantaneous machine configuration of \( M \) after processing the string \( w \) from the previous question. \( [q_3, \lambda] \)
   e) Given string \( w \in \Sigma^* \). Let \( w \in L(M) \). Give the value of the extended transition function, when \( M \) (in the picture) processes \( w \). \( \hat{\delta}(q_0, w) = \quad q_2 \quad \)
   f) Is the DFA \( M \) in the picture completely deterministic? (yes/no) yes

2. (2 points: 0.5 point each) Circle the correct answer.
   a) second argument of the transition function \( \delta \) of a DFA is an element of the set
      \[
      Q \quad Q \cup \{ \lambda \} \quad P(Q) \quad \Sigma \quad \Sigma^* \quad \Sigma \cup \{ \lambda \} \quad F
      \]
   b) the result of the transition function \( \delta \) of a DFA is an element of the set
      \[
      Q \quad Q \cup \{ \lambda \} \quad P(Q) \quad \Sigma \quad \Sigma^* \quad \Sigma \cup \{ \lambda \} \quad F
      \]
   c) second argument of the extended transition function \( \hat{\delta} \) of a DFA is an element of the set
      \[
      Q \quad Q \cup \{ \lambda \} \quad P(Q) \quad \Sigma \quad \Sigma^* \quad \Sigma \cup \{ \lambda \} \quad F
      \]
   d) the result of the extended transition function \( \hat{\delta} \) of a DFA is an element of the set
      \[
      Q \quad Q \cup \{ \lambda \} \quad P(Q) \quad \Sigma \quad \Sigma^* \quad \Sigma \cup \{ \lambda \} \quad F
      \]
3. (1 point) The set  \( \{ w \in \Sigma^* \mid [q_0, w] \xrightarrow{\star} [q_f, \lambda], \text{ where } q_f \in F \} \) is called the __language___ of the DFA  \( M = (Q, \Sigma, \delta, q_0, F) \).

4. (1 point) Given the following incomplete deterministic DFA over the alphabet \{a,b\}:

\[ M: \]

\[ a \quad a \quad b \quad a \quad b \quad a, b \]

\[ q_{error} \]

Make the machine completely deterministic (make additions to the diagram).

5. (1 point) Given a DFA  \( M = (Q, \Sigma, \delta, q_0, F) \). The language of \( M \) is \( L(M) \). Define the machine \( M_1 \) such that \( L(M_1) = \overline{L(M)} \) (complement of \( L(M) \)).

\[ M_1 = (Q, \Sigma, \delta, q_0, \overline{F}) \].

Note that \( \overline{F} \) is the complement of \( F \) with respect to \( Q \).

6. (1 point) Specify the language of a completely deterministic DFA \( M \) with an alphabet \( \Sigma \), whose all states are final states. \( L(M) = \Sigma^* \).

7. (1 point) Specify the language of a DFA \( M \) with alphabet \( \Sigma \), that doesn’t have any final states. \( L(M) = \emptyset \).